Review of Canonical Formalism in HIC

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Contents

1 Applications

- Idea: anti-He^3 productions in p-p collisions

$$\sim \exp\left(-\frac{M}{T}\right) \times \left[V \int \exp\left(-\frac{E_N}{T}\right)\right]^3 \tag{1}$$

(He^3 mass: M, nucleon energy: E_N)

- Free parameters: T, μ_B and V (γ_S).

- Parameters determined by conservation law: μ_Q (no μ_S!).

1.1 Particle productin in low energy HIC

Canonical description works at SIS/AGS energies.

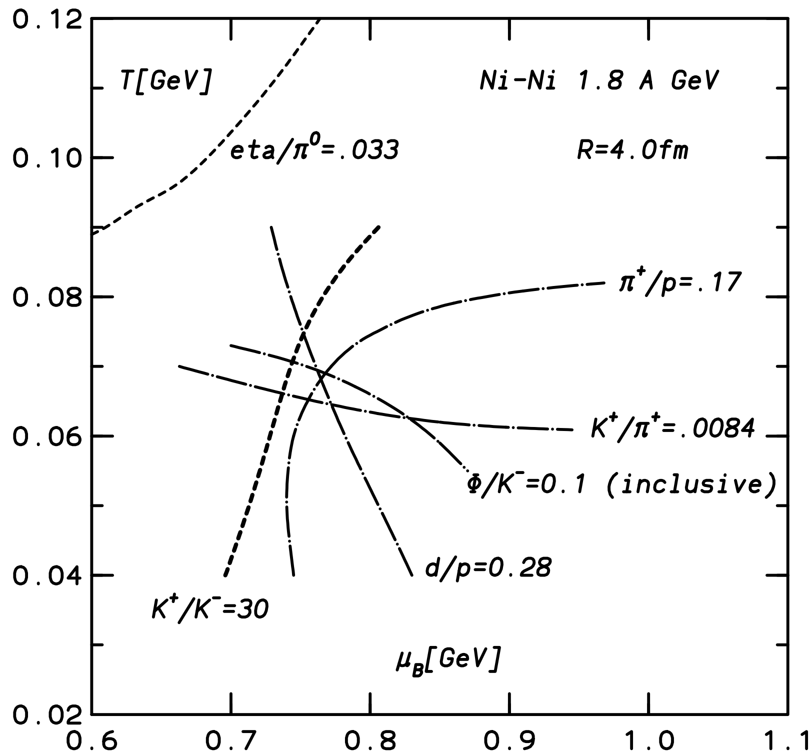

Figure 1: 1.8 A GeV Ni-Ni collision particle ratio lines on the $T - \mu_B$ plane in statistical model.

1.2 Interpretation for strangeness production

QGP or canonical suppression?

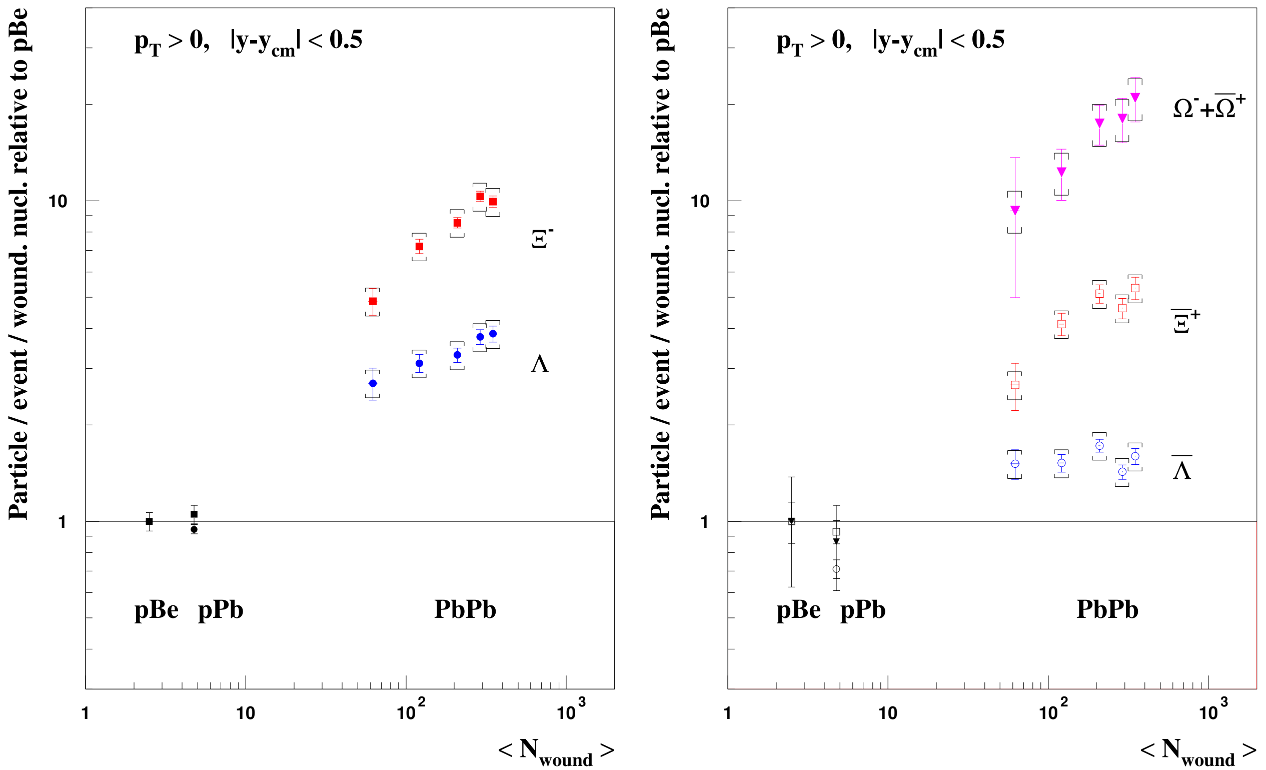

Figure 2: strangeness enhancement.

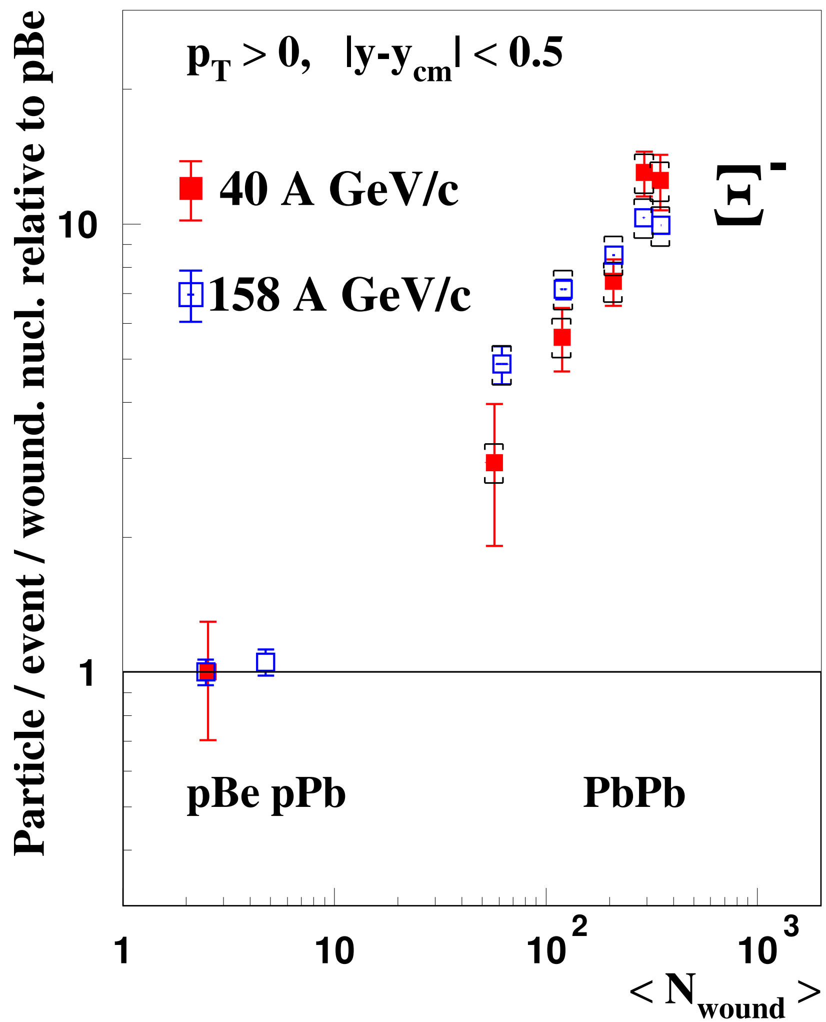

Figure 3: canonical suppression?

2 Formalism

2.1 Grand canonical formalism

- For particle i of strangeness S_i, baryon number B_i, electric charge Q_i and spin–isospin degeneracy factor g_i,

$$\ln Z_i(T,V,\vec{\mu}) = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\beta\epsilon_i)], \tag{2}$$

with (+) for fermions, (-) for bosons and fugacity $\lambda_i(T,\vec{\mu}) = \exp(\frac{B_i\mu_B+S_i\mu_S+Q_i\mu_Q}{T})$

- Grand canonical partition function

$$\ln Z^{GC} = \sum_i \ln Z_i \tag{3}$$

- density of particle i

$$\begin{aligned} n_i^{GC}(T,\vec{\mu}) &= \lambda_i \frac{\partial}{\partial \lambda_i}\ln Z^{GC} \\ &= \frac{T g_i}{2\pi^2}\sum_{k=1}^{\infty}\frac{(\pm 1)^{k+1}}{k}\lambda_i^k m_i^2 K_2(\frac{k m_i}{T}), \end{aligned} \tag{4}$$

2.2 Canonical descriptions

- Grand canonical partition function

  $Z^{GC} = \Sigma_{s=-\infty}^{\infty} \mathrm{tr}_s[e^{-\beta\hat{H}}](\lambda_S)^s$ with $\lambda_S = e^{\beta\mu_S}$.

- Inverse transformation

  $Z_s = \frac{1}{2\pi i} \oint \frac{d\lambda_s}{(\lambda_S)^{s+1}} Z^{GC}(\lambda_S, T, V) = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-is\phi} \tilde{Z}(\phi, T, V)$.

- Canonical partition function with strangeness s

$$Z_s = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \; e^{-is\phi + \left(\Sigma_{n=-3}^{3} S_n e^{in\phi}\right)}, \tag{5}$$

where $S_n = \Sigma_k Z_k^1$ and the sum is over all particles and resonances that carry strangeness n.

- With $s = 0$ and without multistange hadrons, the number density of (anti)strange particle i

$$n^C_{S_i=\pm 1} = n^{GC}_{S_i=\pm 1} I_1(z)/I_0(z) \tag{6}$$

where $z = 2\sqrt{S_1 S_{-1}}$.

3 Check: Kinetic Equation

- Consider $a + b \leftrightarrow c\bar{c}$.

- Definitions

  P_{N_c}: prob. to find N_c particles c.

  G/V: transition prob. per unit time due to productions.

  L/V: transition prob. per unit time due to absorbtions.

- Master equations

$$\begin{aligned} \frac{dP_{N_c}}{d\tau} &= \frac{G}{V} < N_a >< N_b > P_{N_c-1} + \frac{L}{V}(N_c+1)^2 P_{N_c+1} \\ &- \frac{G}{V} < N_a >< N_b > P_{N_c} - \frac{L}{V} N_c^2 P_{N_c} \end{aligned} \tag{7}$$

- With $< N_c >= \Sigma_{N_c=0}^{\infty} N_c P_{N_c}$

$$\frac{d < N_c >}{d\tau} = \frac{G}{V} < N_a >< N_b > - \frac{L}{V} < N_c^2 > \tag{8}$$

- Abundant c: $< N_c^2 > \approx < N_c >^2$

$$< N_c >= \sqrt{\epsilon} \tanh(\tau/\tau_0) \tag{9}$$

where $\sqrt{\epsilon} \equiv \frac{G}{L} < N_a >< N_b >$ and $\tau_0 \equiv \frac{V}{L\sqrt{\epsilon}}$

- rare c: $< N_c^2 > \approx < N_c >$

$$< N_c >= \epsilon(1 - e^{-\tau/\tau_c}) \tag{10}$$

where $\tau_c \equiv V/L$

- With generating functional $g(x,\tau) = \Sigma_{N_c=0}^{\infty} x^{N_c} P_{N_c}(\tau)$

$$\frac{\partial g}{\partial \tau} = \frac{L}{V}(1-x)(x\frac{\partial^2 g}{\partial x^2} + \frac{\partial g}{\partial x} - \epsilon g) \tag{11}$$

- At equilibrium

$$g_{eq}(x) = \frac{I_0(2\sqrt{\epsilon x})}{I_0(2\sqrt{\epsilon})} \tag{12}$$

$$P_{N_c,eq} = \frac{\epsilon^{N_c}}{(N_c!)^2 I_0(2\sqrt{\epsilon})} \tag{13}$$

$$< N_c >_{eq} = \sqrt{\epsilon}\frac{I_1(2\sqrt{\epsilon})}{I_0(2\sqrt{\epsilon})} \tag{14}$$

4 Equivalent to GC?

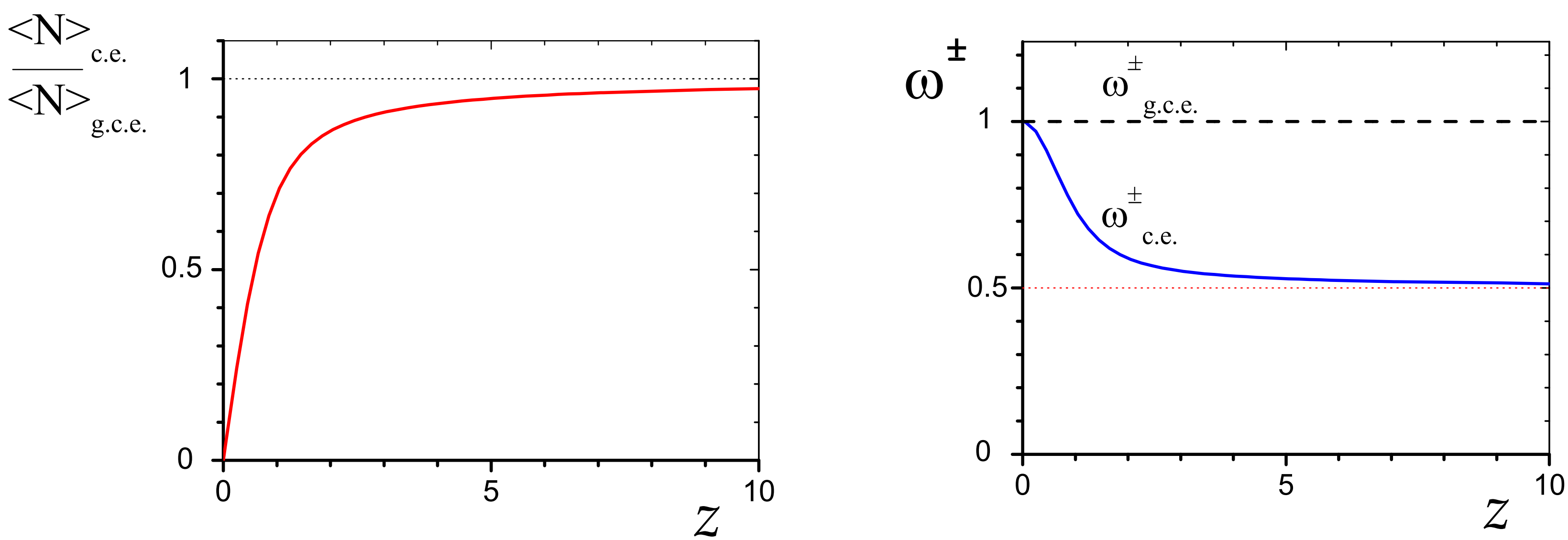

Figure 4: Canonical vs. Grand Canonical