Statistical Model for SIS/AGS with In-Medium Masses

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Contents

1 Motivations

We know that

- Grand Canonical Statistical Model: Good from AGS to RHIC (in Braun-Munzinger's talk).
- For the low energy/small systems: canonical statistical model.
- Free-space masses as parameters

What if modium modifications ($m \rightarrow m^*, \cdots$) are incorporated ?

- Krakow group (00,01)

  Pb-Pb collisions at SPS (GC model).

  Assume that universal scaling except for Goldstone bosons.

  20% mass change allowed.

  In-medium effects of kaons? Of excited/heavy quark hadrons?

  Krakow group's scaling looks too universal.

- Frankfurt-Argonne group (02,04)

  AGS to RHIC (GC model)

  SU(3) $\sigma - \omega$ model

  Similar figures as free cases

  No scaling of vector and scalar?

  Pion's mass scales more than kaon's?

- Our work

  Proper scaling nature especially for kaons

  Low Energy Heavy Ion Collisions

2 Canonical Model with conserved charges

2.1 Simple grand canonical (GC) formalism

- For particle i of strangeness S_i, baryon number B_i, electric charge Q_i and spin–isospin degeneracy factor g_i,

$$\ln Z_i(T, V, \vec{\mu}) = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\epsilon_i/T)], \tag{1}$$

with (+) for fermions, (-) for bosons and fugacity

$$\lambda_i(T, \vec{\mu}) = \exp\left((B_i\mu_B + S_i\mu_S + Q_i\mu_Q)/T\right) \tag{2}$$

- density of particle i

$$n_i(T, \vec{\mu}) = \frac{\langle N_i \rangle}{V} = \frac{T g_i}{2\pi^2} \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \lambda_i^k m_i^2 K_2(\frac{k m_i}{T}), \tag{3}$$

- μ_Q and μ_S are determined by their conservation laws.
- Free parameters: T and baryon chemical potential μ_B
- It works well for the chemical compositions in heavy ion collisions.

2.2 Canonical formalism

- Hagedorn's example: the number density of $\overline{\mathrm{He}^3}$ in pp collisions:

$$
\begin{aligned}
n_{GC} &\sim \exp\left(-\frac{M_{He3}}{T}\right), \\
n_{C} &\sim \exp\left(-\frac{M_{He3}}{T}\right) \times \left[V \int d^3p \exp\left(-\frac{E_N}{T}\right)\right]^3,
\end{aligned} \tag{4}
$$

where M_{He3}: He^3 mass, $E_N = \sqrt{m_N^2 + \vec{p}^2}$: nucleon's energy. $n_{GC}/n_C \sim 10^7$!

- Grand canonical partition function

$$
Z^{GC} = \sum_{s=-\infty}^{\infty} \mathrm{tr}_s[e^{-\beta \hat{H}}](\lambda_S)^s \tag{5}
$$

with $\lambda_S = e^{\beta \mu_S}$

- Inverse transformation to choose a fixed s

$$
Z_s = \frac{1}{2\pi i} \oint \frac{d\lambda_s}{(\lambda_S)^{s+1}} Z^{GC}(\lambda_S, T, V) \tag{6}
$$

- Canonical partition function with fixed $s = 0$

$$Z^{C}_{s=0} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \, \exp\left(\sum_{n=-3}^{3} S_n e^{in\phi} \right), \tag{7}$$

where $S_n = \Sigma_k Z^1_k$ and the sum is over all particles and resonances that carry strangeness n.

- In the thermodynamic limit, $C \to GC$ in number densities, not in fluctuations.

- At SIS/AGS energies, canonical ensemble is necessary for the strangeness.

- In the canonical treatment, we do *not* have μ_S, but we have dependence on the volume, V.

3 In-Medium Masses for our model

- Nuclear density is defined as

$$\rho = \frac{2}{\pi^2} \int dp \frac{p^2}{e^{(\sqrt{k^2+m_N^{*2}}-\mu+V_n)/T} + 1}. \tag{8}$$

- Recent BR study about the temperature dependence of the low energy scaled mesons

  To 125 MeV: The mass dropping rate is about 5

  After 125 MeV to T_c: The mass drops to zero

  $(1 - T^2/T_c^2)^{1/10}$ mimics such behaviors.

- The density dependence of scaling ratio is approximated by

$$(1 - 0.2\rho/\rho_0) \tag{9}$$

- We assume that strange constituent quark mass and the excitation energies from the low-lying states do NOT scale.

- Nucleons:

$$\frac{m_N^*}{m_N} = \Phi(\rho, T), \quad \Phi(\rho, T) = (1 - 0.2\rho/\rho_0)(1 - T^2/T_c^2)^{1/10} \tag{10}$$

with $T_c = 200$ MeV and $V_n = 270\rho/\rho_0$ MeV.

- Other baryons (and baryonic resonances):

$$m_B^* = m_B - \frac{N_l}{3} m_N \left[1 - \Phi(\rho, T)\right] \tag{11}$$

where N_l is the numbers of u and d quarks in the baryon. And the all of them feels the repulsion

$$V = \frac{270 N_l}{3} \frac{\rho}{\rho_0} \tag{12}$$

- Vectors : Brown-Rho scaling for ρ, ω and $\sigma(700)$.

$$\frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = \frac{m_\sigma^*}{m_\sigma} = \Phi(\rho, T). \tag{13}$$

- $m_\phi * = m_\phi$. No light quark.

- K^*:

$$m^*_{K^*} = \frac{1}{2}\left[m_\rho \Phi(\rho, T) + m_\phi\right] \tag{14}$$

which represent the sliding light quark part and the non-sliding strange part.

- π and η : chiral symmetry breaking is small, and their masses are not expected to change much. We put $m^*_{\pi,\eta} = m_{\pi,\eta}$.

- Kaons. Large change in m_K due to large Σ_{KN} and the Wess-Zumino term.

$$m^*_K = \left(\frac{m_K^2 - \rho\Sigma/f_\pi^2}{1 - 0.37\frac{\rho\Sigma_{KN}}{f_\pi^2(m_K \mp V_K)^2}}\right)^{1/2} \left(1 - \frac{T^2}{T_c^2}\right)^{1/10}, \tag{15}$$

where $m_K = 494$ MeV, $f_\pi = 93$ MeV, $\Sigma_{KN} = 400$ MeV and $V_k = 90\rho/\rho_0$ MeV which is also added(subtracted) in the energy of $K^-, \bar{K}_0(K^+, K_0)$. $(-)$ corresponds to K^- and $\bar{K}^0$ and $(+)$ corresponds to their antiparticles.

- η': We assume that U(1) anomaly contribution melt in the same way as chiral condensate,

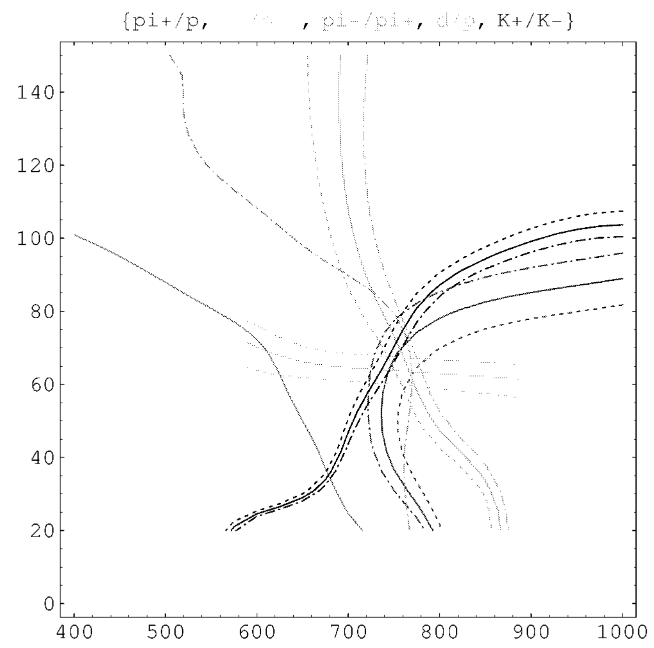
$$m^*_{\eta'} = 283 * \Phi(\rho, T) + 700. \tag{16}$$

- $a_0(980)$ and $f_0(980)$ are highly excited and maybe tetraquarks so we put their masses unchanged.
- We include particles that have more than 3 stars in PDG data book. The baryon below 1.80GeV, the meson below 1.1GeV, the antibaryon below 1 GeV are included.

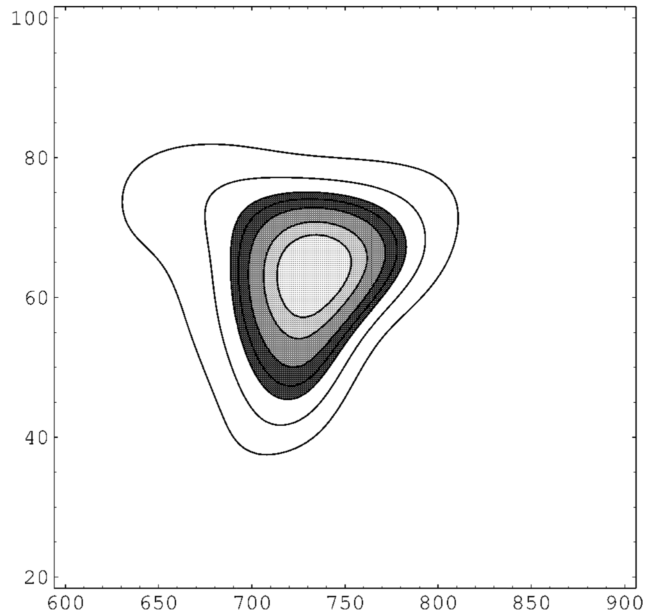
4 Preliminary Results

4.1 Ni+Ni at SIS

- With Free masses (R=4fm)
- Freeze out point: T =63.7 MeV, μ =732 MeV, $\chi^2 = 1.2$

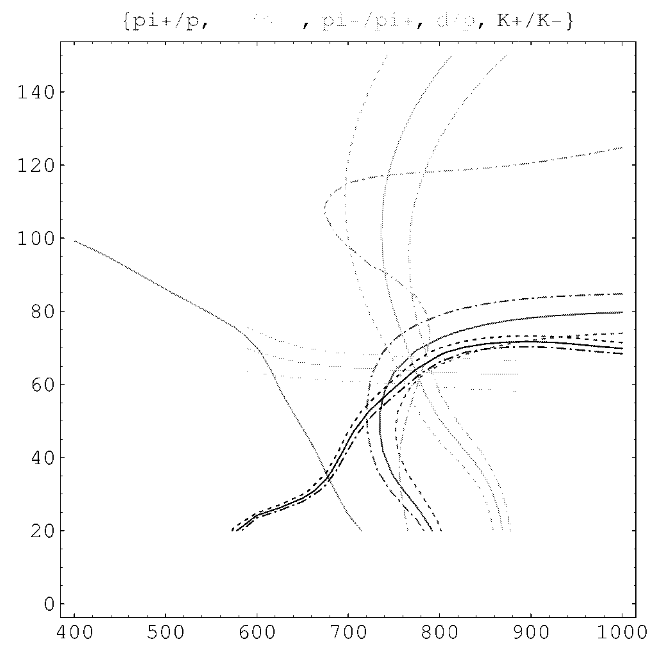


- 1.8 A GeV Ni-Ni collision particle ratio lines on the $T - \mu_B$ plane with free masses

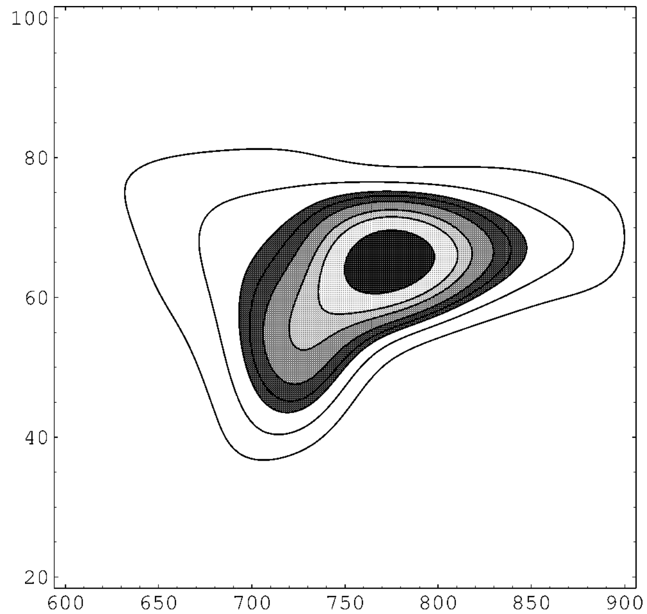


- 1.8 A GeV Ni-Ni collision χ^2 contours with free masses

- With in-medium masses
- Freeze out point: T =65.3 MeV, μ =773 MeV, $\chi^2 = 0.46$



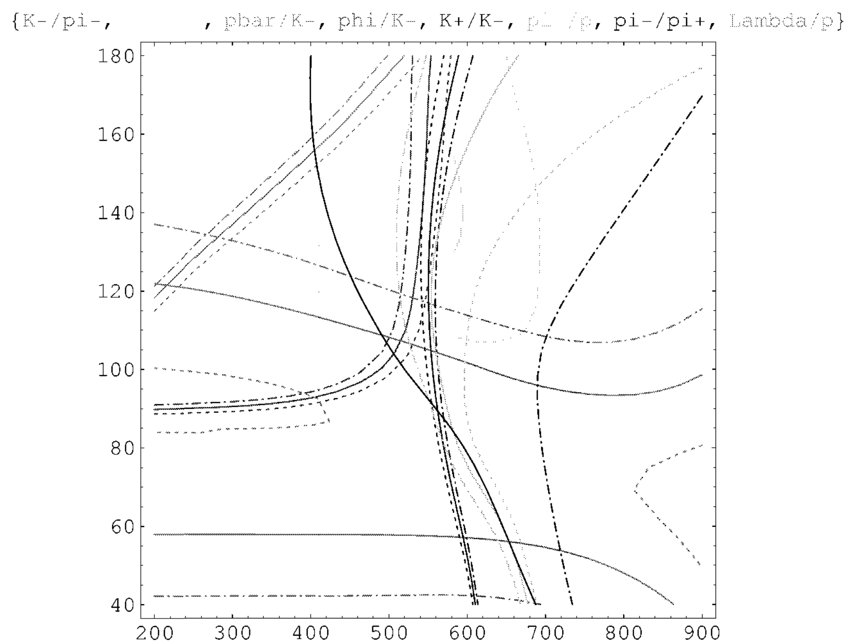
• 1.8 A GeV Ni-Ni collision particle ratio lines on the $T - \mu_B$ plane with in-medium masses



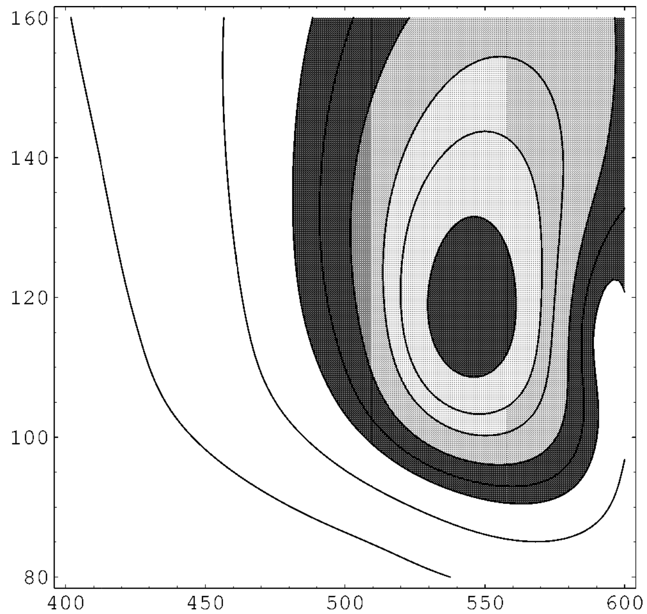
- 1.8 A GeV Ni-Ni collision χ^2 contours with in-medium masses

4.2 Si+Au at AGS

- With Free masses (R=4.9fm)
- Freeze out point: T =118 MeV, μ =545 MeV, $\chi^2 = 0.54$

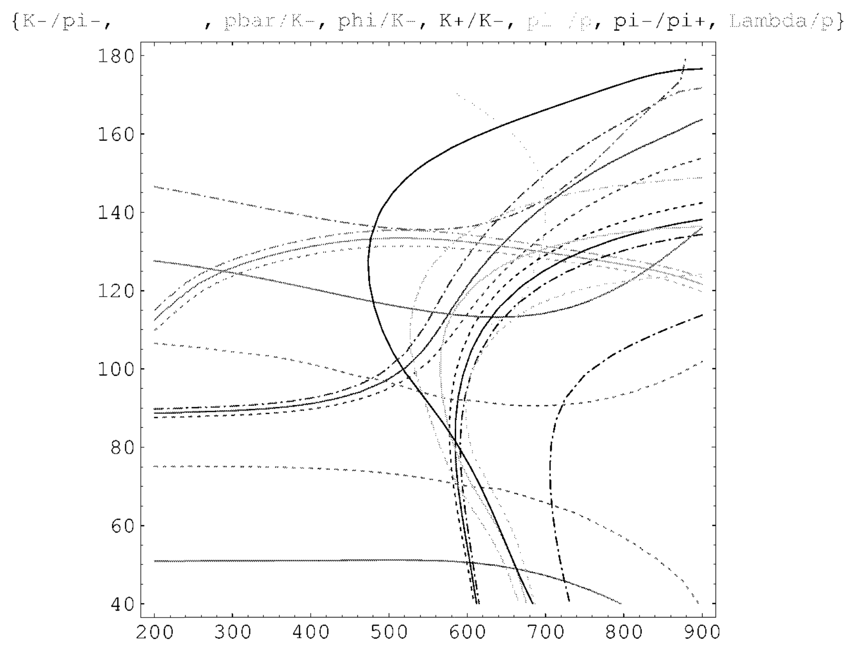


- 14.6 A GeV Si-Au collision particle ratio lines on the $T - \mu_B$ plane with free masses

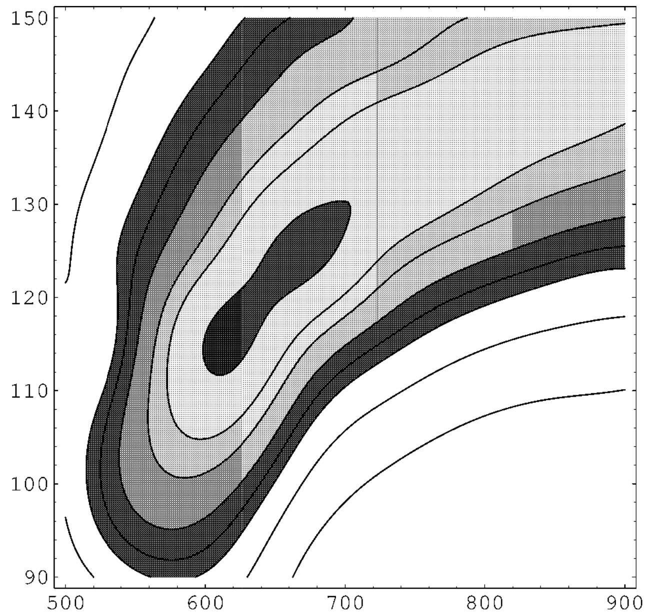


- 14.6 A GeV Si-Au collision χ^2 contours with free masses

- With in-medium masses
- Freeze out point: T =116 MeV, μ =617 MeV, $\chi^2 = 0.83$



- 14.6 A GeV Si-Au collision particle ratio lines on the $T - \mu_B$ plane with in-medium masses



- 14.6 A GeV Si-Au collision χ^2 contours with in-medium masses

5 Conclusions

- Our results are preliminary yet and we will study 2-10 GeV A case. (Lack of exact data)
- We can obtain the common chemical freeze out points at SIS/AGS energy range with the in-medium massess considered here.
- With our in-medium effects the freezeout chemical potential becomes larger. (Not lowered)
- With our in-medium effects the freezeout temperature is not so much changed. (Increasing at SIS, Decreasing at AGS)