Gauged nonlinear sigma model in AdS_5 space and hadron physics

Xiao-Hong Wu

Korea Institute for Advanced Study

Collaborated with Y. Kim and P. Ko, in preparation.

Outline

- AdS/CFT correspondence
  dictionary, holographic model to describe the low energy ρ, a_1, π resonance dynamics, chiral symmetry breaking (spontaneously and explicitly)

- summary of the results without dim-6 operators
  the problem: KSRF relation, vanishing $\Gamma(a_1 \to \pi\gamma)$, D/S ratio of process $a_1 \to \rho\pi$, pion charge radius r_π

- our model with 2 dim-6 operators
  $i\kappa \mathrm{Tr}\Big[L_{MN} D^M \Phi D^N \Phi^\dagger + R_{MN} D^M \Phi^\dagger D^N \Phi\Big]$, $\qquad \zeta \mathrm{Tr}\Big[L_{MN} \Phi R^{MN} \Phi^\dagger\Big]$

- spectrum and decay constant
  m_ρ, m_{a_1}, f_π, f_ρ and f_{a_1}

- interactions and numerical results
  $a_1 \to \rho\pi$, $\rho \to \pi\pi$, $a_1 \to \pi\gamma$, pion electromagnetic form factor, chiral coefficients L_i

- summary

AdS/CFT correspondence

- AdS/CFT correpondence
  In the large N and large t' Hooft coupling $\lambda = g^2_{\rm YM} N$ limit, type IIB string theory on $\mathrm{AdS}_5 \times S^5$ space is dual to $\mathcal{N} = 4$ SU(N) 4D super Yang-Mills theory.

$$\int \mathcal{D}\phi_{CFT} \; e^{-S_{CFT}[\phi_{CFT}] - \int d^4x \;\phi_0 \mathcal{O}} = \int_{\phi_0} \mathcal{D}\phi \; e^{-S_{bulk}[\phi]} \equiv e^{iS_{eff}[\phi_0]}$$

$$\langle \mathcal{O} \ldots \mathcal{O} \rangle = \frac{\delta^n S_{eff}}{\delta\phi_0 \ldots \delta\phi_0} \qquad (\text{for n} - \text{point functions})$$

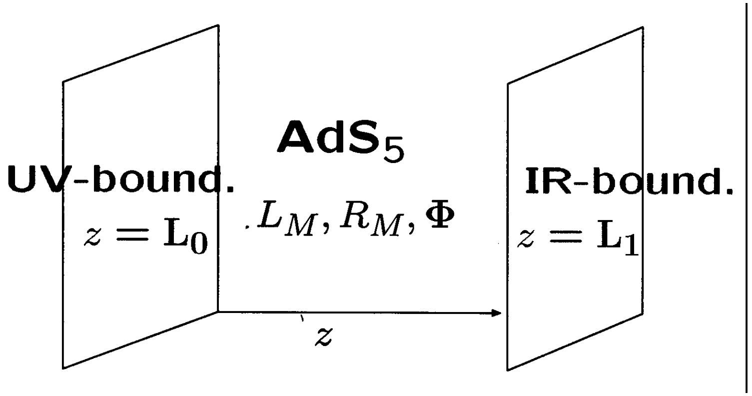
- dictionary
  4D CFT $\Longleftrightarrow$ 5D AdS
  global symmetry in CFT $\Longleftrightarrow$ gauge symmetry in bulk
  operator $\mathcal{O}(x)$ $\Longleftrightarrow$ bulk field $\Phi(x, z)$
  dimension of $\mathcal{O}(x)$ $\Longleftrightarrow$ bulk mass of $\Phi(x, z)$
  large N_c $\Longleftrightarrow$ gauge coupling $M_5 L$
  large momentum Q $\Longleftrightarrow$ small z

Holographic QCD model

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, hep-ph/0501128; L. Da Rold and A. Pomarol, hep-ph/0501218

- chiral symmetry $SU(3)_L \otimes SU(3)_R$
  dual to gauge symmetry in AdS$_5$ with metric $ds^2 = (\frac{L}{z})^2(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$
- color singlet flavor operators and low energy dynamics of ρ, a_1, π
  $\bar{q}^i_R q^j_L \Longleftrightarrow 3 \times 3$ scalar Φ, $(\bar{3}, 3)$
  $\bar{q}_L \gamma^\mu t^a q_L \Longleftrightarrow$ left gauge field L^a_M, $(8, 0)$
  $\bar{q}_R \gamma^\mu t^a q_R \Longleftrightarrow$ right gauge field R^a_M, $(0, 8)$

$$\mathcal{L}_5 = \sqrt{g} M_5 \mathrm{Tr}\Big[-\frac{1}{4} L_{MN} L^{MN} - \frac{1}{4} R_{MN} R^{MN} + \frac{1}{2} D_M \Phi^\dagger D^M \Phi - \frac{1}{2} M^2_\Phi \Phi^\dagger \Phi \Big]$$



$L_0 \to 0$ (UV), and L_1 (IR), breaks scale invariance and gives a mass gap of order $\Lambda_{\rm QCD}$.

- chiral symmetry breaking, $SU(3)_L \otimes SU(3)_R \Longrightarrow SU(3)_V$
  $\langle \bar{q} q \rangle$, quark mass $m_q \Longleftrightarrow \langle \Phi(x, z) \rangle$
  $[\bar{q}q] = 3 \Longleftrightarrow M^2_\Phi = \Delta(\Delta - 4) = -3$
  solution $\langle \Phi \rangle = M_q z + \xi \frac{z^3}{L_1^3}$, describe chiral symmetry breaking both spontaneously and explicitly.

Holographic QCD model (cont'd)

- mass spectrum and decay constants
  KK decomposition $\Phi(x,z) = \sum_n f_n(z)\Phi^{(n)}(x)$,
  identify ρ, a_1 and π as the lowest resonance.
  then the problem reduces to two-boundary problem with proper boundary condition.
  calculate m_ρ, f_ρ, m_{a_1}, f_{a_1} f_π
  interaction vertex, $g = \int_{L_0}^{L_1} dz f_i(z) f_j(z) f_k(z)$

case f_π	L_1 ξ	κ (10^{-6}) ζ (10^{-6})	m_ρ f_ρ	m_{a_1} f_{a_1}	$\Gamma(\rho \to \pi\pi)$ $g_{\rho\pi\pi}$	$\Gamma(a_1 \to \pi\gamma)$ r_π(fm)	$\Gamma(a_1 \to \rho\pi)$ D/S ratio
exp 86.4 ± 9.7			775.8 ± 0.5	1230 ± 40	146.4 ± 1.5	0.640 ± 0.246 0.66 ± 0.02	$250 \sim 600$ -0.108 ± 0.016
A 85.0	3.125 4.0	0. 0.	769.6 138	1253 163	95.4 4.8	0. 0.585	295.5 -0.055

Tab. 1: The unit of masses, decay constants and decay widths is MeV.

- the problem
  wrong KSRF relation $\frac{g^2_{\rho\pi\pi}}{m^2_\rho} = \frac{1}{3f^2_\pi}$ (3 $\to$ 2), small $g_{\rho\pi\pi}$, vanishing $\Gamma(a_1 \to \pi\gamma)$, small D/S ratio in $a_1 \to \rho\pi$ decay, small pion charge radius r_π.

Model with dim-6 operators

- We expect higher dimension operators can contribute to $a_1 \to \pi\gamma$ and improve the phenomenology, such as pion charge radius, D/S ratio in $a_1 \to \rho\pi$ decay.

$$\mathcal{L}_5^{\mathrm{dim}-6} = \sqrt{g} M_5 \,\mathrm{Tr}\Big[- i \frac{\kappa}{M_5^2} \Big(L_{MN} D^M \Phi D^N \Phi^\dagger + R_{MN} D^M \Phi^\dagger D^N \Phi \Big) + \frac{\zeta}{M_5^2} L_{MN} \Phi R^{MN} \Phi^\dagger \Big]$$

$$V_M = \frac{1}{\sqrt{2}} (L_M + R_M), \qquad A_M = \frac{1}{\sqrt{2}} (L_M - R_M)$$

- similiar operators are considered in 4D models.
  in the framework of massive YM theory, U. -G. Meissner, Phys.Rept.161(1988) 213.
  linear sigma model, P. Ko, S. Rudaz, Phys.Rev.D50 (1994) 6877.

Mass spectra and decay constants

- two-point correlator

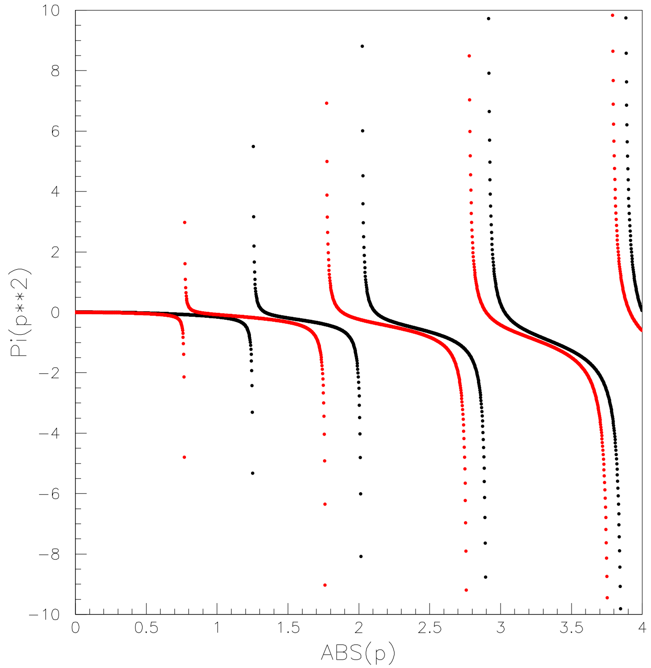
$$\Pi(p^2) = M_5 L \frac{\partial_5 f(z)}{z f(z)}\Big|_{z=L_0\to 0}$$

with $f(z)$ as the solution of equation of motion with two-boundary condition.

- In large N_c limit, correlator is related with the resonance masses and decay constants

$$\Pi_A(p^2) = p^2 \sum_n \frac{f_{A_n}^2}{p^2 - M_{A_n}^2} + f_\pi^2$$

$$\Pi_V(p^2) = p^2 \sum_n \frac{f_{V_n}^2}{p^2 - M_{V_n}^2}$$



- mass as the pole, decay constants related with the residue

$$f_{\rho,a_1}^2 = \lim_{p^2\to m_{\rho,a_1}^2} (p^2 - m_{\rho,a_1}^2)\Pi_{V,A}(p^2)/p^2$$

$$f_\pi^2 = \Pi_A(0)$$

$a_1 \to \rho\pi$

- $a_1 \to \rho\pi$, 3 Lorents structure

$$\mathcal{L}_{a_1\rho\pi} = ig_{1a_1\rho\pi}\mathrm{Tr}(\tilde{A}^{\mu}[\tilde{V}_{\mu}, \tilde{A}_z]) + ig_{2a_1\rho\pi}\mathrm{Tr}(\tilde{A}^{\mu}[\tilde{V}^{\mu\nu}, \partial_{\nu}\tilde{A}_z]) + ig_{3a_1\rho\pi}\mathrm{Tr}(\tilde{A}^{\mu\nu}[\tilde{V}^{\mu\nu}, \tilde{A}_z])$$

$g_{1a_1\rho\pi}$: dim-4, κ term, ζ term
$g_{2a_1\rho\pi}$: κ term
$g_{3a_1\rho\pi}$: ζ term
D/S wave amplitudes

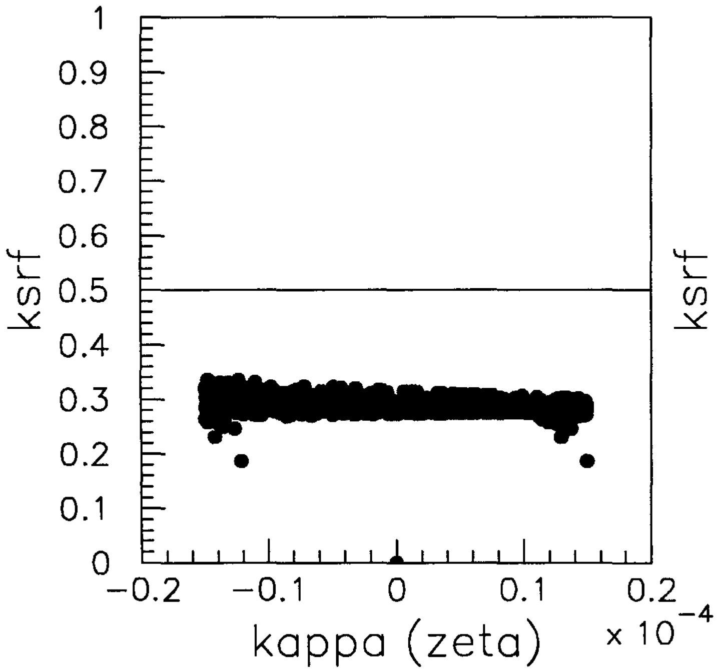
$$\langle \rho(\vec{k}s_{\rho})\pi(-\vec{k})|H|a_1(0s_{a_1})\rangle = if^S_{a_1\rho\pi}\delta_{s_{\rho}s_{a_1}}Y_{00}(\Omega_k) + if^D_{a_1\rho\pi}\sum_{m_L} C(211; m_L s_{\rho} s_{a_1})Y_{2m_L}(\Omega_k)$$

$\rho \to \pi\pi$

- $\rho \to \pi\pi$, with both minimal and non-minimal coupling

$$\mathcal{L}_{\rho\pi\pi} = \frac{i}{\sqrt{2}} g_{\rho\pi\pi} \mathrm{Tr}(\tilde{V}^{\mu}[\tilde{A}_5, \partial_\mu \tilde{A}_5]) + \frac{i}{\sqrt{2}} f_{\rho\pi\pi} \mathrm{Tr}(\tilde{V}^{\mu\nu}[\partial_\mu \tilde{A}_5, \partial_\nu \tilde{A}_5])$$

$g_{\rho\pi\pi}$: dim-4, κ term, ζ term
$f_{\rho\pi\pi}$: κ term
KSRF relation: $\frac{g^2_{\rho\pi\pi}}{m^2_\rho} = \frac{1}{2f^2_\pi}$



- check the low energy theorem, $\mathcal{O}(p^2)$ four-pion interactions

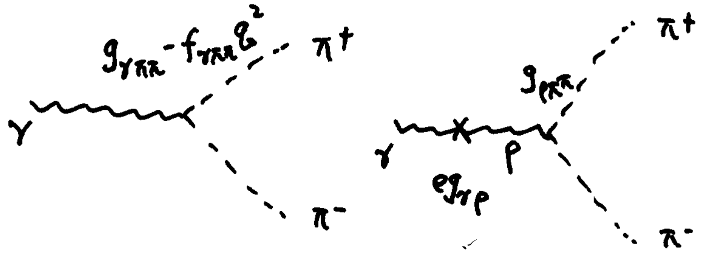
$$g_{\pi^4} + \sum_n \frac{g^2_{n\pi\pi}}{M^2_{\rho_n}} = \frac{1}{3f^2_\pi}$$

Interactions: $\gamma\pi\pi$ vertex

- photon as external field
  compared with Da Rold and Pomarol's method to introduce photon as U(1) subgroup of $SU(3)_V$.
  no need to worry about photon KK states, calculate pion charge radius r_π

$$V_\mu(x,z) = eF_\mu(x)\tau_3 + \frac{1}{\sqrt{M_5 L}} \sum_{n=1}^{\infty} \tilde{V}_\mu^{(n)}(x) f_V^{(n)}(z)$$

- pion electromagnetic form factor
  kinetic mixing of γ and ρ, $\mathcal{L}_{\gamma\rho} = -\frac{1}{2} e g_{\gamma\rho} F^{\mu\nu} \tilde{V}_{\mu\nu}$
  pion electromagnetic form factor, in the small momentum limit
  $F(q^2) = 1 + \frac{1}{6} r_\pi^2 q^2 + \mathcal{O}(q^4)$
  pion charge radius r_π, $r_\pi^2 = 6\left[-\frac{f_{\gamma\pi\pi}}{g_{\gamma\pi\pi}} + \frac{g_{\gamma\rho} g_{\rho\pi\pi}}{m_\rho^2} \right]$



- different from usual VMD
  kinetic mixing or $\gamma - \rho$ mixing mass term

$a_1 \to \pi\gamma$

- $a_1 \to \pi\gamma$, similar to $a_1 \to \rho\pi$

$$\mathcal{L}_{a_1\rho\pi} = ig_{1a_1\rho\pi}\mathrm{Tr}(\tilde{A}^\mu[\tilde{V}_\mu, \tilde{A}_z]) + ig_{2a_1\rho\pi}\mathrm{Tr}(\tilde{A}_\mu[\tilde{V}^{\mu\nu}, \partial_\nu \tilde{A}_z]) + ig_{3a_1\rho\pi}\mathrm{Tr}(\tilde{A}^{\mu\nu}[\tilde{V}_{\mu\nu}, \tilde{A}_z])$$

We have checked the non-gauge invariant term $\mathrm{Tr}(\tilde{A}^\mu[\tilde{V}_\mu, \tilde{A}_z])$ is cancelled. Then only κ and ζ term contribute to $a_1 \to \pi\gamma$.

- As reference, the generalized hidden local symmetry
$a_1 \to \pi\gamma$ and $a_1 \to \rho\pi$ have the same Lorentz structure, $\mathrm{Tr}(\tilde{A}^{\mu\nu}[\tilde{V}_{\mu\nu}, \pi])$,
$\Gamma(a_1 \to \pi\gamma) : \Gamma(a_1 \to \rho\pi) \sim e^2/g^2_{\rho\pi\pi}$
M. Bando, T. Fujiwara, K. Yamawaki, Prog.Theor.Phys.79:1140,1988.

numerical results

- case A (without dim-6 operators),
  case B (fit m_ρ, m_{a_1}, D/S ratio, $\Gamma(\rho \to \pi\pi)$),
  case E (fit m_ρ, m_{a_1}, $\Gamma(\rho \to \pi\pi)$, f_π).

case f_π	L_1 ξ	κ (10^{-6}) ζ (10^{-6})	m_ρ f_ρ	m_{a_1} f_{a_1}	$\Gamma(\rho \to \pi\pi)$ $g_{\rho\pi\pi}$	$\Gamma(a_1 \to \pi\gamma)$ r_π(fm)	$\Gamma(a_1 \to \rho\pi)$ D/S ratio
exp 86.4 ± 9.7			775.8 ± 0.5 ~ 160	1230 ± 40	146.4 ± 1.5	0.640 ± 0.246 0.672 ± 0.008	$250 \sim 600$ -0.108 ± 0.016
A 85.0	3.125 4.0	0. 0.	769.6 138	1253 163	95.4 4.8	0. 0.585	295.5 -0.055
B 71.9	2.836 2.56	-5.930 -39.72	775.8 144	1230 182	146.5 5.8	0.088 0.654	165.3 -0.094
E 78.7	3.102 4.010	-16.03 0.09188	775.8 140	1246 172	146.4 5.6	0.042 0.640	409.8 -0.027

Tab. 2: The unit of masses, decay constants and decay widths is MeV.

- non-vanishing $\Gamma(a_1 \to \pi\gamma)$, still 2σ away
- pion charge radius r_π, good agreement
- KSRF relation $\frac{g_{\rho\pi\pi}^2}{m_\rho^2} = \frac{1}{2f_\pi^2}$, $(2 \to 3)$, still bad

chiral coefficients

- chiral coefficients, L_3: also from S, $L_{4,5,6}$: S, L_7: P, L_8: S, the $\mathcal{O}(p^4)$ chiral Lagrangian

$$
\begin{aligned}
\mathcal{L}_4 &= L_1 \mathrm{Tr}^2[D_\mu U^\dagger D^\mu U] + L_2 \mathrm{Tr}[D_\mu U^\dagger D_\nu U]\mathrm{Tr}[D^\mu U^\dagger D^\nu U] + L_3 \mathrm{Tr}[D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U] \\
&+ L_4 \mathrm{Tr}[D_\mu U^\dagger D^\mu U]\mathrm{Tr}[U^\dagger \chi + \chi^\dagger U] + L_5 \mathrm{Tr}[D_\mu U^\dagger D^\mu U(U^\dagger \chi + \chi^\dagger U)] \\
&+ L_6 \mathrm{Tr}^2[U^\dagger \chi + \chi^\dagger U] + L_7 \mathrm{Tr}^2[U^\dagger \chi - \chi^\dagger U] + L_8 \mathrm{Tr}[\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi] \\
&- i L_9 \mathrm{Tr}[F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U] + L_{10} \mathrm{Tr}[U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}]
\end{aligned}
$$

- chiral coefficients, integrate out ρ and a_1

$$
\begin{aligned}
L_1 &= \frac{f_\pi^4}{8m_\rho^4} g_{\rho\pi\pi}^2 - \frac{f_\pi^4}{4m_\rho^4} g_{\rho\pi\pi} f_{\rho\pi\pi}, \qquad L_2 = 2L_1, \qquad L_3 = -6L_1, \\
L_9 &= \frac{f_\pi^4}{m_\rho^4} g_{\rho\pi\pi}^2 + \frac{f_\pi^2}{2m_\rho^2} e g_{\gamma\rho} g_{\rho\pi\pi} - \frac{2f_\pi^4}{m_\rho^2} g_{\rho\pi\pi} f_{\rho\pi\pi}, \qquad L_{10} = \frac{1}{4}[\Pi_A'(0) - \Pi_V'(0)]
\end{aligned}
$$

- electromagnetic pion mass difference, operator $\mathrm{Tr}[Q_R U Q_L U^\dagger]$

$$
m_{\pi^+} - m_{\pi^0} \simeq \frac{3\alpha_{\mathrm{em}}}{8\pi m_\pi f_\pi^2} \int_0^\infty dp^2 (\Pi_A - \Pi_V)
$$

case	L_1	L_2	L_3	L_9	L_{10}	$m_{\pi^+} - m_{\pi^0}$ (MeV)
exp	0.4 ± 0.3	1.4 ± 0.3	-3.5 ± 1.1	6.9 ± 0.7	-5.5 ± 0.7	4.6
A	0.43	0.86	-2.6	5.1	-5.5	3.4
B	0.32	0.65	-1.9	4.0	-5.0	1.5
E	0.46	0.93	-2.8	5.3	-5.1	2.9

Tab. 3: The unit of chiral coefficients L_i is 10^{-3}.

Summary

- We considered dim-6 operator contributions in holographic QCD model.
  ρ, a_1 mass spectra, decay constants, phenomenology, chiral coefficients

- non-vanishing $\Gamma(a_1 \to \pi\gamma)$, still 2σ away

- pion charge radius, good agreement
  different VMD

- KSRF relation, not good

  M. Harada, S. Matsuzaki, K. Yamawaki, hep-ph/0603248.