McLerran-Venugopalan Model in the Heavy-Ion Collision

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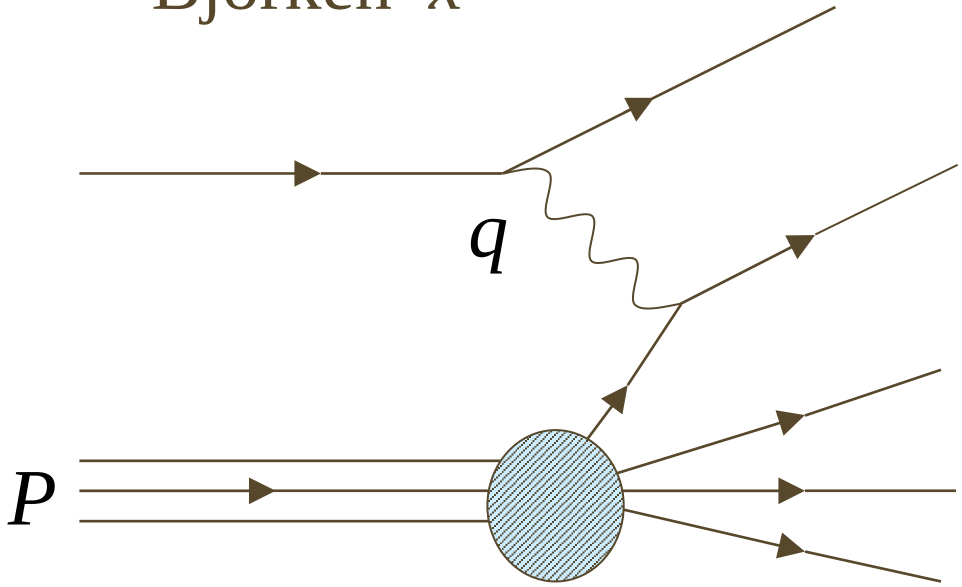
Things to be discussed

- What is the MV model?
- Why is it so hard analytically?
- How is it solved numerically?
- What is the problem?
- Conclusion
  - – Venugopalan, Krasnitz, Nara, Lappi ... ???
  - – Venugopalan, Romatchke, Lappi ... ???
- Hard to give the correct initial condition...

Frequently Used Variables

- Two Variables (in DIS)
  - Virtuality Q^2
  - Bjorken x



$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{s + Q^2 + M^2}$$

small-x $x << 1$

⇕

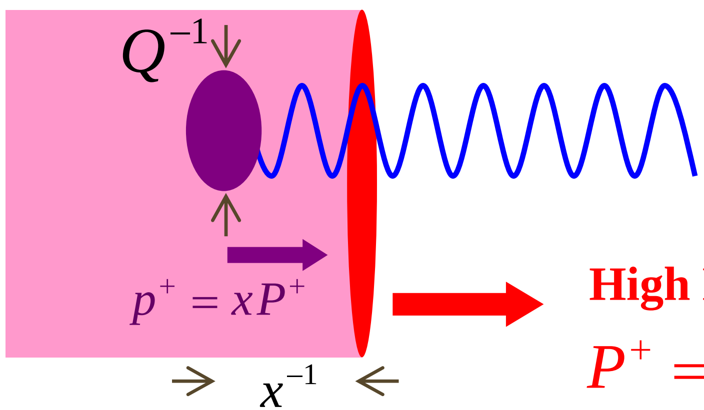
high energy $s >> Q^2$

Convenient Interpretation

- Intuitive Meaning of Two Variables
  - Transverse Momentum Q^2

    Transverse size of partons
  - Bjorken x

    Longitudinal fraction of parton momentum



Light Projectile (photon if DIS)

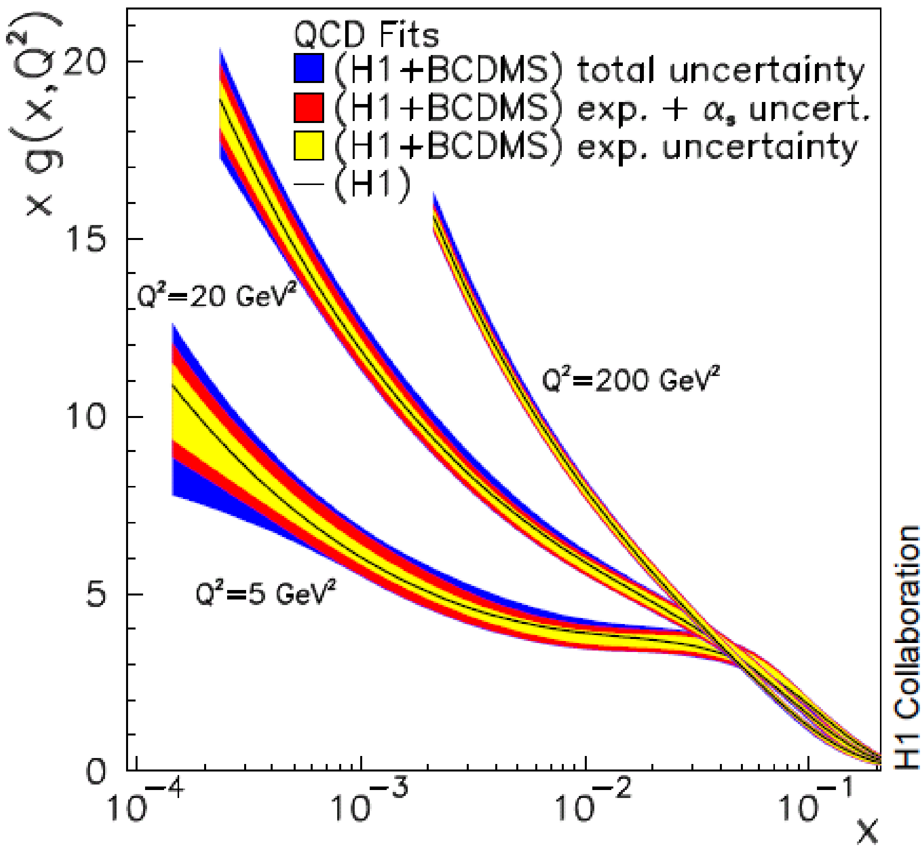
$$q^\mu = (q_0, q_\perp, 0)$$

High Energy Hadron Target

$$P^+ = \frac{1}{\sqrt{2}}(P^0 + P^z) \sim \infty$$

Gluon Evolution

- Parton (Gluon) distribution grows up



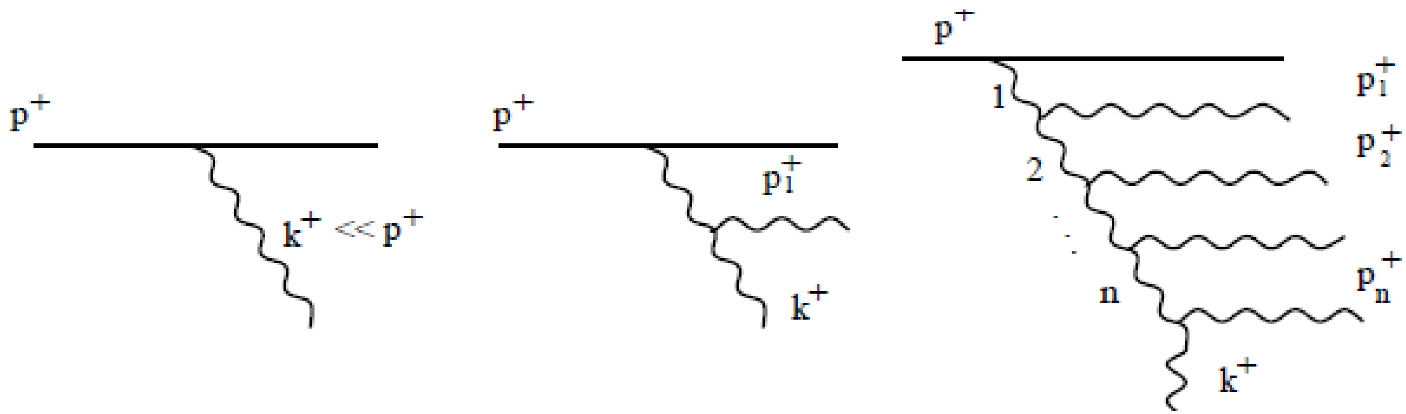
as *x* goes smaller
BFKL dynamics

or

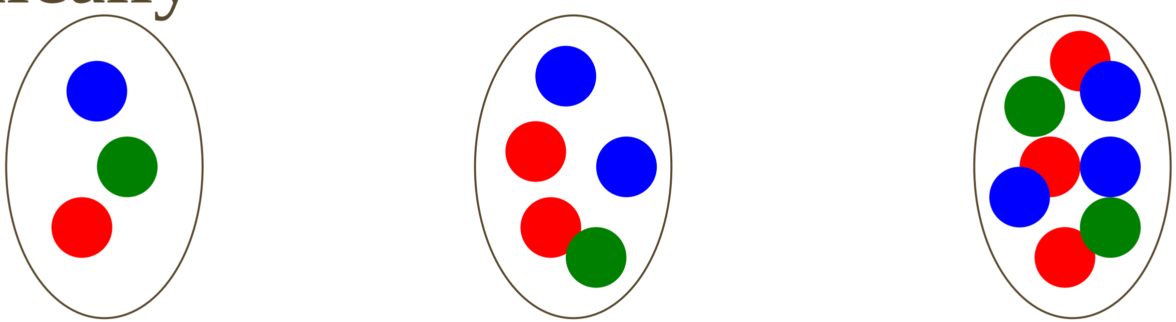
as *Q* goes larger
DGLAP dynamics

Going to smaller x with fixed Q

- Gluon increases with a fixed transverse area



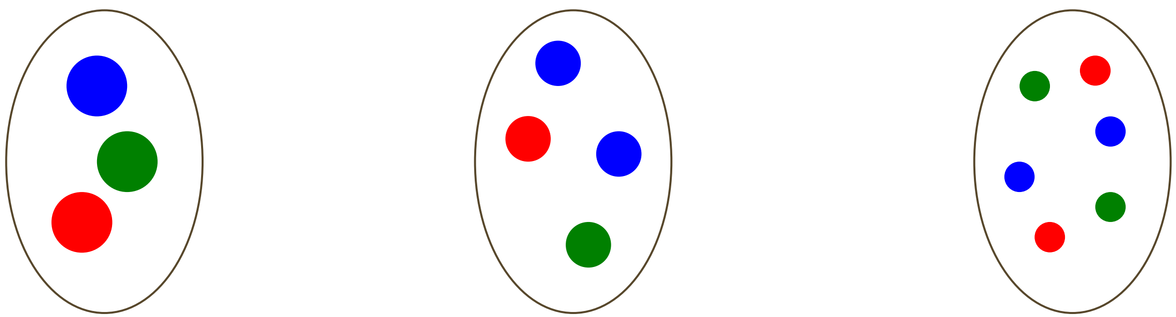
- Graphically



small-*x* → Dense Gluon Matter

Going to larger Q^2 with fixed x

- Gluon slowly increases with a decreasing area
- Graphically, in the same way,



large Q → Dilute Gluon Matter

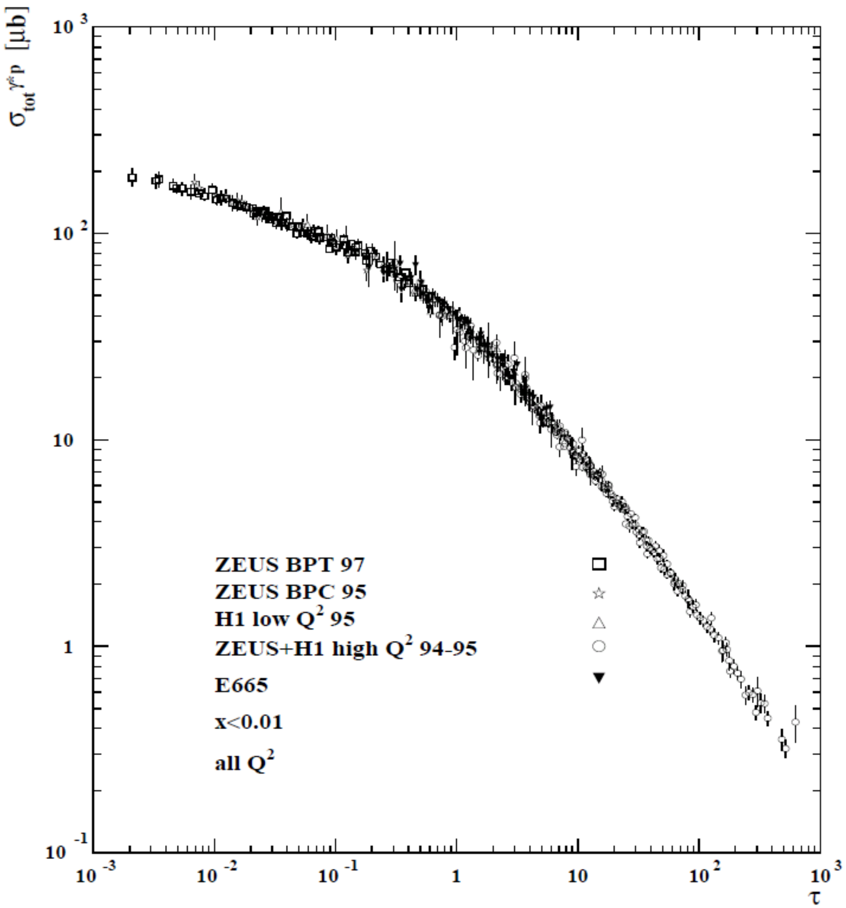
- When does the distribution come to overlap?

$$xg(Q_s, x) \Big/ (N_c^2 - 1) \cdot Q_s^2 \cdot \pi R_A^2 \sim 1$$

Gluons with $k_t << Q_s(x)$ are saturated.

Saturated, then, Simple!

- x and Q not independent but…



$$\sigma_{\gamma^* p}(x, Q^2) \to \sigma_{\gamma^* p}(Q^2 / Q_s^2(x))$$

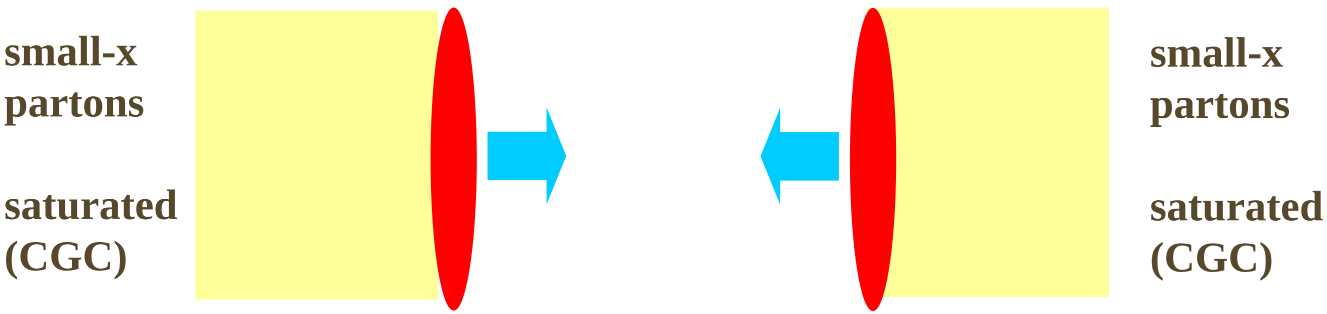
$$Q_s^2(x) = Q_0^2 (x / x_0)^{-\lambda}$$

Called the Geometric Scaling

Stasto-Golec-Biernat-Kwiecinski Plot

Nucleus-Nucleus Collisions

- Particles $p_t < 1\text{GeV}$



$$x \sim p_t \big/ \sqrt{s} \sim 10^{-2} \quad (\sqrt{s} = 200\text{GeV})$$

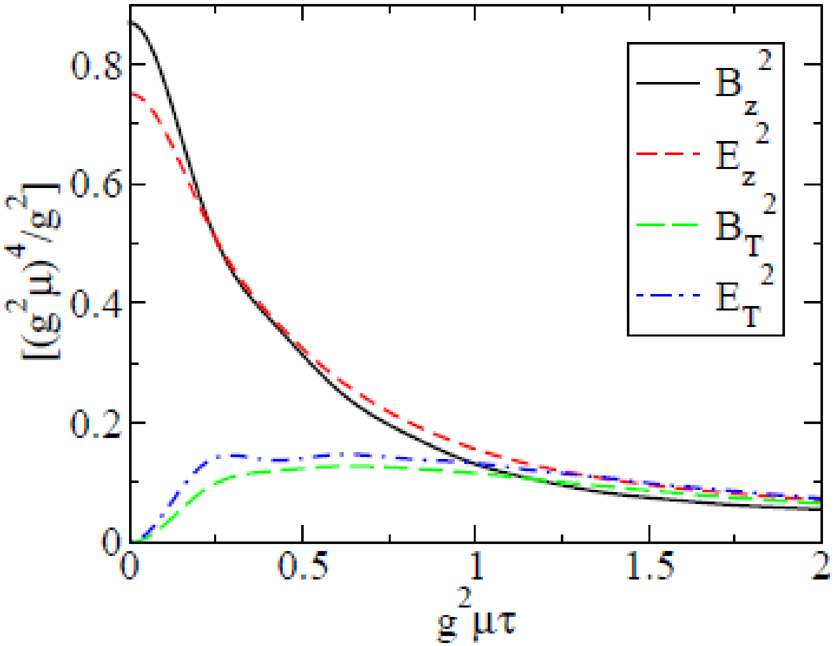
$$Q_s = Q_0 (x_0 / x)^{\lambda} \cdot A^{1/6} \sim 1\text{-}2\,\text{GeV}$$

Initial Time-Evolution scales as $\sim \tau Q_s$
Initial Energy-Density proportional to $Q_s{}^4$

Fries-Kapusta-Li ('06)
K.F. ('07)

Some (Wrong) Numerical Results

- Longitudinal *E* and *B* fields
- Topological number



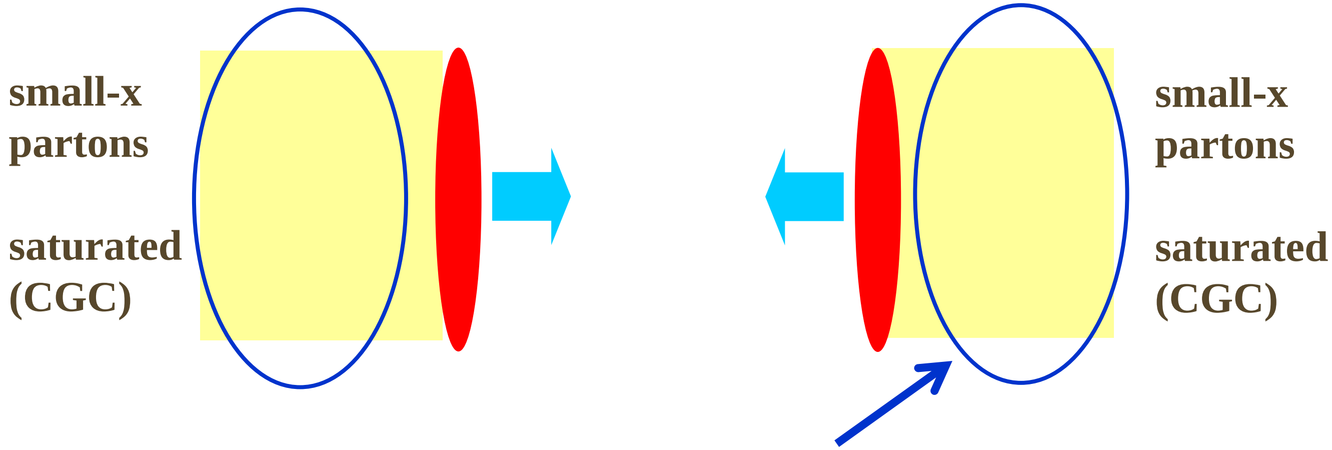
Color Glass Condensate melting to Quark-Gluon Plasma

GLASMA

Lappi-McLerran ('05)

Coherent Gluon Fields

- Equation of motion leads to...



Fields are given as a function of nucleus source

$$\mathcal{A}_i = -\frac{1}{ig} V(x_\perp, x^-)\, \partial_i V^+(x_\perp, x^-)$$

$$V^+(x_\perp, x^-) = P_{x^-} \exp\left[-ig \int_{-\infty}^{x^-} dz^- \frac{1}{\partial_\perp^2} \rho(x_\perp) \delta(z^-) \right]$$

Saturation Scale in the MV Model

- Physical observables as a function of

$$V^{+}(x_\perp, x^-) = P_{x^-} \exp\left[-ig \int_{-\infty}^{x^-} dz^- \frac{1}{\partial_\perp^2} \rho(x_\perp) \delta(z^-) \right]$$

- Gaussian weight

$$W[\rho] = \exp\left[-\int d^2 x_T dx^- \frac{|\rho(x)|^2}{2g^2 \mu^2 (x^-)} \right]$$

Practical Implementation

- Approximation

$$\bar{V}^{+}(x_\perp) = \exp\left[-ig\,\theta(x^-)\frac{1}{\partial_\perp^2}\rho(x_\perp)\right]$$

- Gaussian weight

$$\bar{W}[\rho] = \exp\left[-\int d^2x_T\,\frac{|\rho(x)|^2}{2g^2\bar{\mu}^2}\right] \qquad \bar{\mu}^2 = \int dx^- \mu^2(x^-)$$

Regularized Expressions

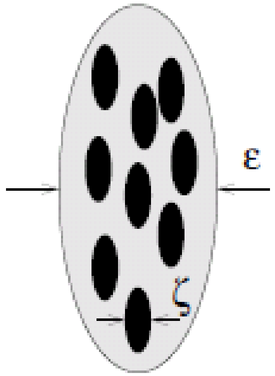
- Longitudinal Extent

$$V_\varepsilon^+(x_\perp, x^-) = P_{x^-} \exp\left[-ig \int_{-\infty}^{x^-} dz^- \frac{1}{\partial_\perp^2} \rho_\varepsilon(x_\perp, z^-) \right]$$

- Randomness

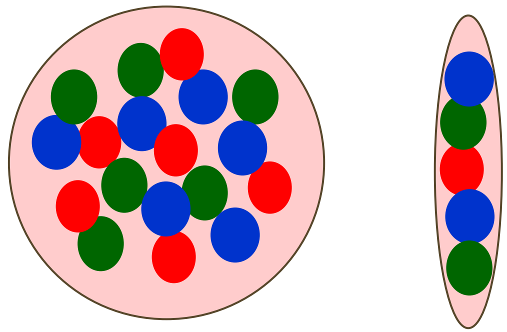
$$\left\langle \rho_a(x_T, x^-) \rho_b(y_T, y^-) \right\rangle_\varsigma = g^2 \mu^2 (x^-) \delta^{(2)}(x_T - y_T) \delta_\varsigma (x^- - y^-)$$

- Schematically →
- Two limits not commutative

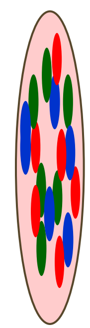


Non-commutative Limits

- Numerical Method
  - Take the $\varepsilon \rightarrow 0$ limit first
  - Take the $\zeta \rightarrow 0$ limit then

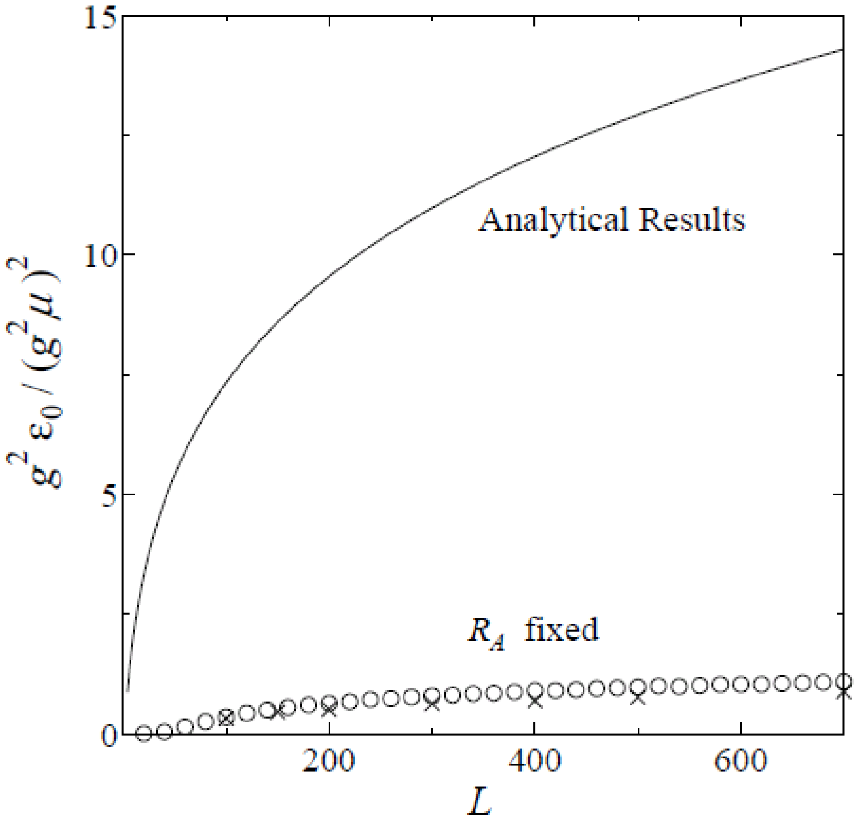


- In reality, we should...
  - Take the $\zeta \rightarrow 0$ limit first
  - Take the $\varepsilon \rightarrow 0$ limit then



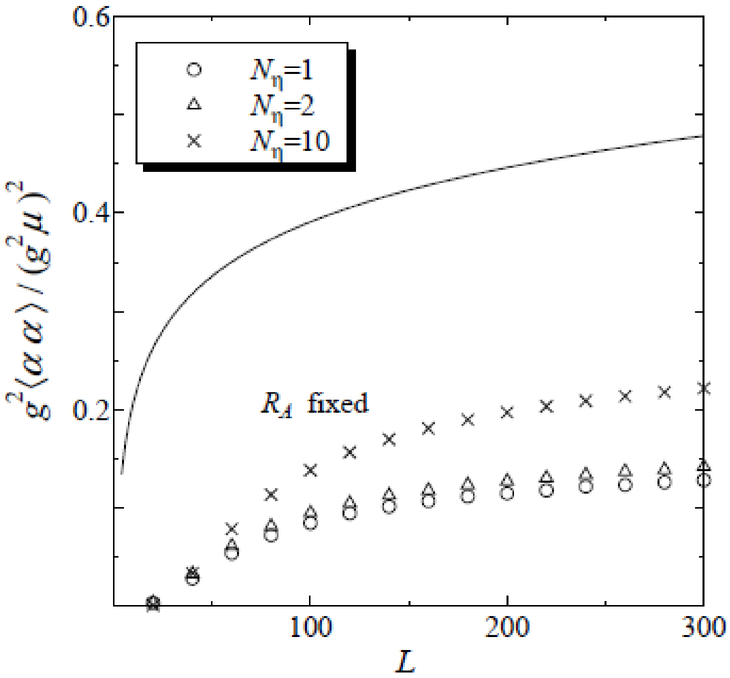
Initial Energy Density

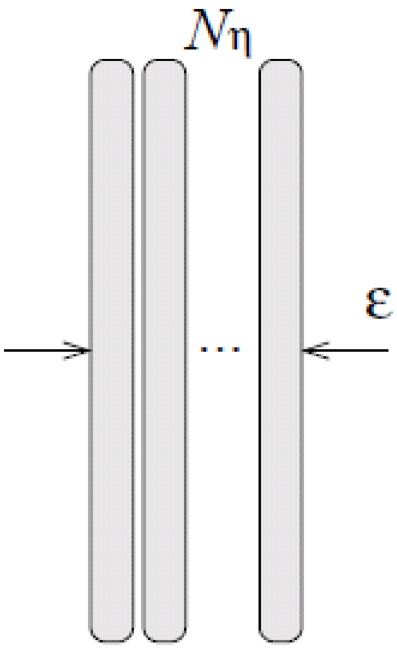
- **Huge Difference!**



How to improve?

- Insert longitudinal slices
- A little better...





Discussions

- Numerical implementation of the MV model assumes an irrelevant order of two limits.
- Energy density underestimates by ~14.
- Then, numerical calculations meaningless???
- Maybe... but μ could rescue them...
- If μ is twice larger, energy density becomes 16 times larger...

A wrong anser with a wrong parameter could give a right answer...