Lattice QCD in
Finite Temperature/Density

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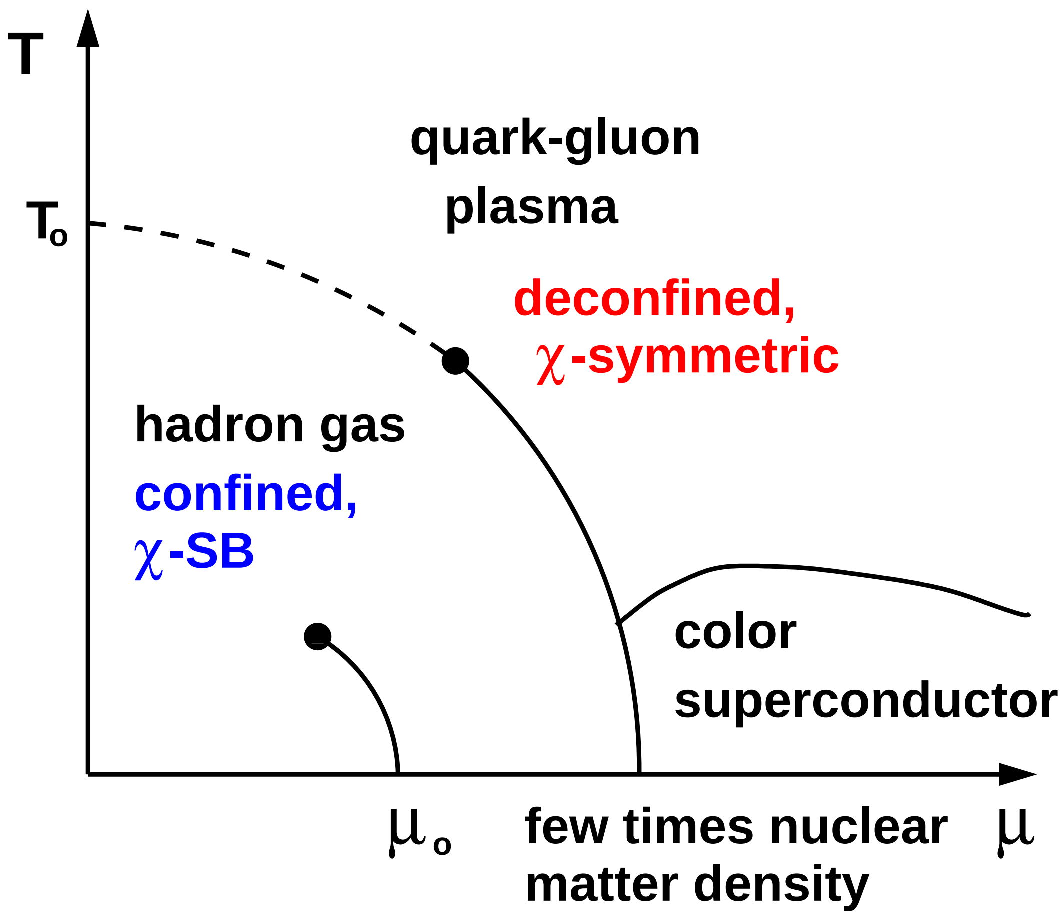
KPS meeting on 070419

http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=32#session-8

Plan of Talk

1. Lattice QCD
2. Lattice QCD in Finite Temperature
3. Lattice QCD with Finite Baryon Density
4. Discussion

1. Lattice QCD



schematic QCD phase diagram

$$\langle \mathcal{O} \rangle = \frac{\int [dA_\mu(x)][d\psi(x)][d\overline{\psi}(x)] e^{-S[A_\mu,\psi,\overline{\psi}]} \mathcal{O}[A_\mu,\psi,\overline{\psi}]}{\int [dA_\mu(x)][d\psi(x)][d\overline{\psi}(x)] e^{-S[A_\mu,\psi,\overline{\psi}]}} \quad (1)$$

- observables are given by the ratio of two integrals
- existence of integral?
- a regulator is necessary
- discretization of space-time
- numerical integration (Monte Carlo method)
- continuum limit ($a \to 0$)
- finite volume effect (large enough spacetime)
- realistic quark mass

- three(more ?) different phases of QCD states
- hadronic phase

$$V(r) = -\frac{\alpha}{r} + \sigma r \tag{2}$$

- quark-gluon plasma (QGP) phase

$$V(r) = -\frac{\alpha}{r}e^{-\mu r} + \frac{\sigma}{\mu}(1 - e^{-\mu r}) \tag{3}$$

- color superconducting phase

2. Lattice QCD in Finite Temperature

- No problem with simulation! it is “just” difficult
- Recent focus on realistic simulations

- Example: RBC-Bielefeld, PRD74, 054507(2006), hep-lat/0608013

- $N_f = 2+1$, p4fat3 staggered quark action, tree-level Symanzik improved gauge action

- $N_t = 4, 6$. $N_s = 8, 16, 24, 32$

- $m_q = 0.00325 \sim 0.05 (N_t = 4), 0.004 \sim 0.016 (N_t = 6)$ $(150 \leq m_\pi \leq 500)\mathrm{MeV}$

- $m_s = 0.01 \sim 0.065 (N_t = 4), 0.04 (N_t = 6)$

- Exact RHMC algorithm

V. THE TRANSITION TEMPERATURE

To obtain the transition temperature we use the results for the scales r_0/a and $\sqrt{\sigma}a$ obtained from fits to the static quark potential. In cases where zero temperature calculations have not been performed directly at the critical coupling but at a nearby β-value we use Eq. (12) to determine the scales at $\beta_c(\hat{m}_l, \hat{m}_s, N_\tau)$. The transition temperature, lattice parameters $(\hat{m}_l, \hat{m}_s, N_\tau)$, to the chiral and continuum limit using an ansatz that takes into account the quadratic cut-off dependence, $(aT)^2 = 1/N_\tau^2$, and a quark mass dependence expressed in terms of the pseudoscalar meson mass,

$$Y_{\hat{m}_l, \hat{m}_s, N_\tau} = Y_{0, m_s, \infty} + A(m_{ps} r_0)^d + B/N_\tau^2, \qquad Y = T_c r_0, \quad T_c/\sqrt{\sigma}, \tag{13}$$

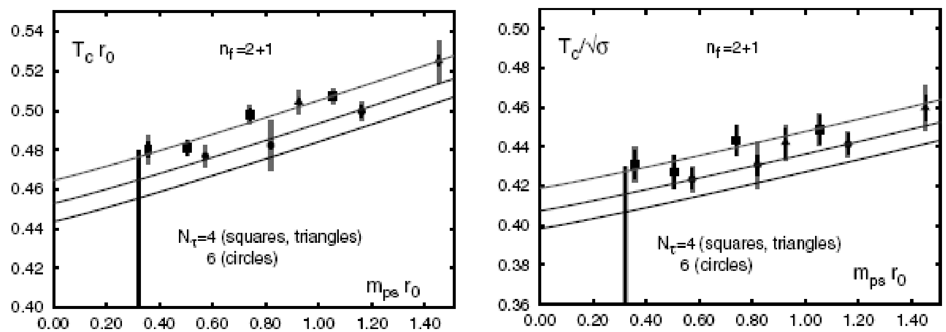


FIG. 7 (color online). $T_c r_0$ (left) and $T_c/\sqrt{\sigma}$ (right) as a function of $m_{ps} r_0$ on lattices with temporal extent $N_\tau = 4$, $\hat{m}_s = 0.065$ (squares) and $\hat{m}_s = 0.1$ (triangles) as well as for $N_\tau = 6$, $\hat{m}_s = 0.04$ (circles). Thin error bars represent the statistical and systematic error on r_0/a and $\sqrt{\sigma}a$. The broad error bar combines this error with the error on β_c. The vertical line shows the location of the physical value $m_{ps} r_0 = 0.321(5)$ and its width represents the error on r_0. The three parallel lines show results of fits based on Eq. (13) with $d = 1.08$ for $N_\tau = 4$, 6 and $N_\tau \to \infty$ (top to bottom).

- $T_c = 192(7)(4)$, rapid cross-over

3. Lattice QCD with Finite Baryon Density

- "Sign problem" with lattice formulation of finite baryon density

- With finite chemical potential QCD lagrangin is complex

$$M(\mu) = \gamma_\mu D_\mu + m + \mu\gamma_0 \quad (4)$$

can't find Λ for $M^\dagger(\mu) = \Lambda M(\mu)\Lambda^{-1}$

- Monte Carlo simulation is difficult (importance sampling may be accomplished only after large cancellation)

• Recently devised methods for small baryon density (small μ, chemical potential) region

(a) Multi-parameter reweighting

(b) Taylor expansion of observables around $\mu = 0$

(c) Imaginary chemical potential method

• For example, critical end point at $(T, \mu_B) \sim (\ 162,\ 360)$ from (2+1 flavor) (a) method

• continuum limit, chiral extrapolation, finite volume effect

- Hydrodynamic quantities: slope of various spectral functions at zero frequency limit

- bulk viscosity, shear viscosity, electric conductivity, etc

- Swansea-Sejong collaboration, hep-lat/0703008

$$G(\tau) = \int d^3x \langle J(\tau,\vec{x}) J^\dagger(0,\vec{0})\rangle = \int_0^\infty \frac{d\omega}{2\pi} K(\omega,\tau)\rho(\omega) \tag{5}$$

$$K(\omega,\tau) = \frac{\cosh[\omega(\tau - \frac{1}{2}T)]}{\sinh(\frac{\omega}{2T})} \tag{6}$$

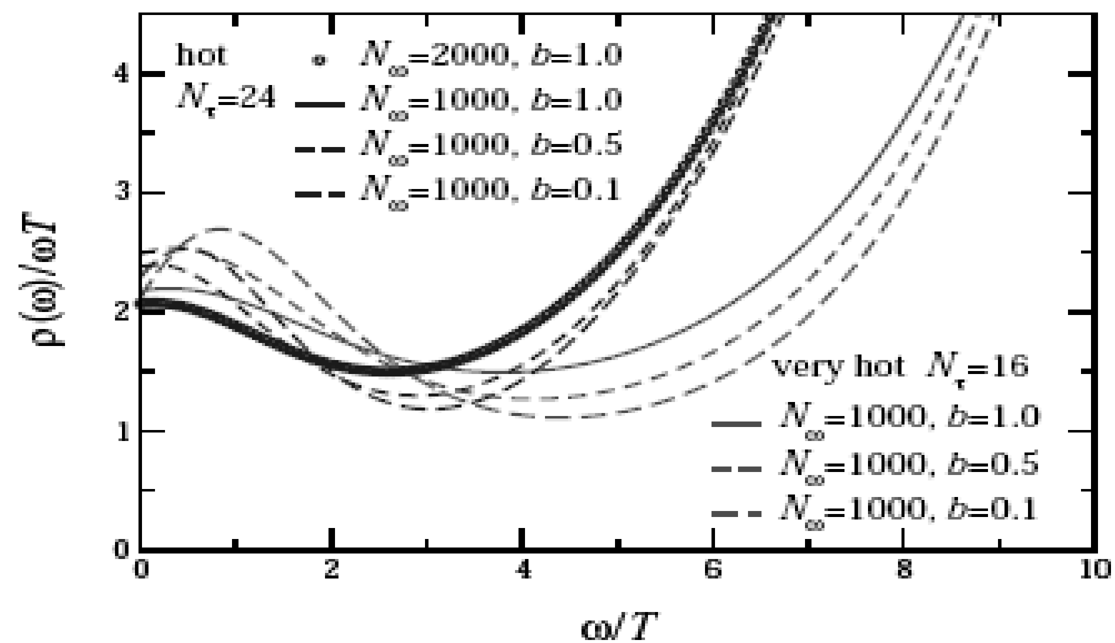
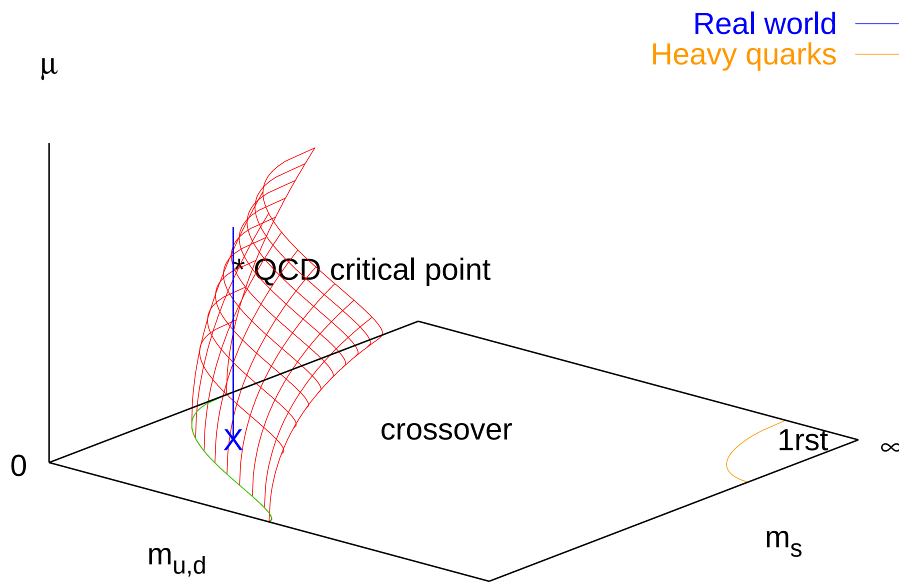


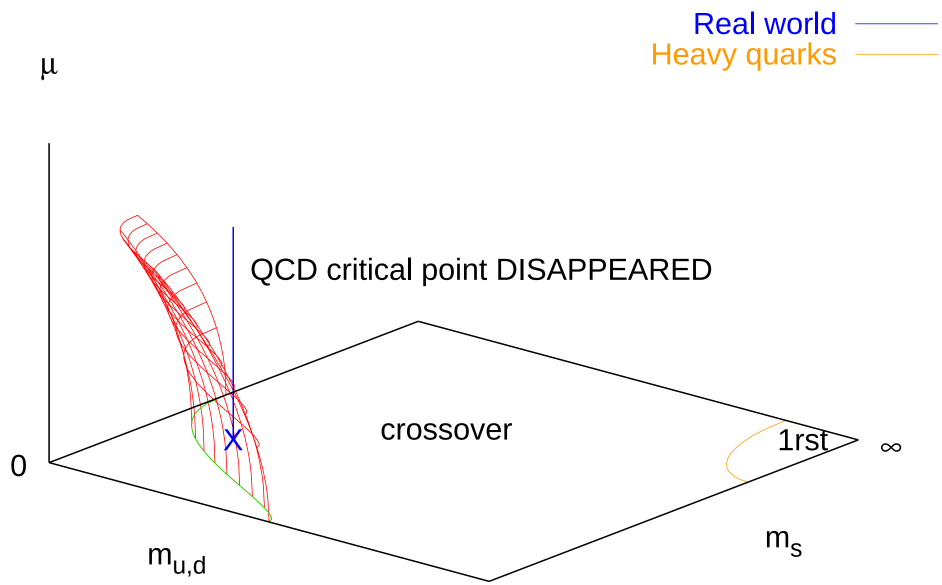
FIG. 4: Default model dependence of $\rho(\omega)/\omega T$ for N_τ = 24 (hot) and 16 (very hot) in the low-energy region. We show results for $N_\omega = 1000, 2000$ and $b = 1.0, 0.5, 0.1$ at fixed $a\omega_{\max} = 5$.

- $\sigma/T = 0.4 \pm 0.1$

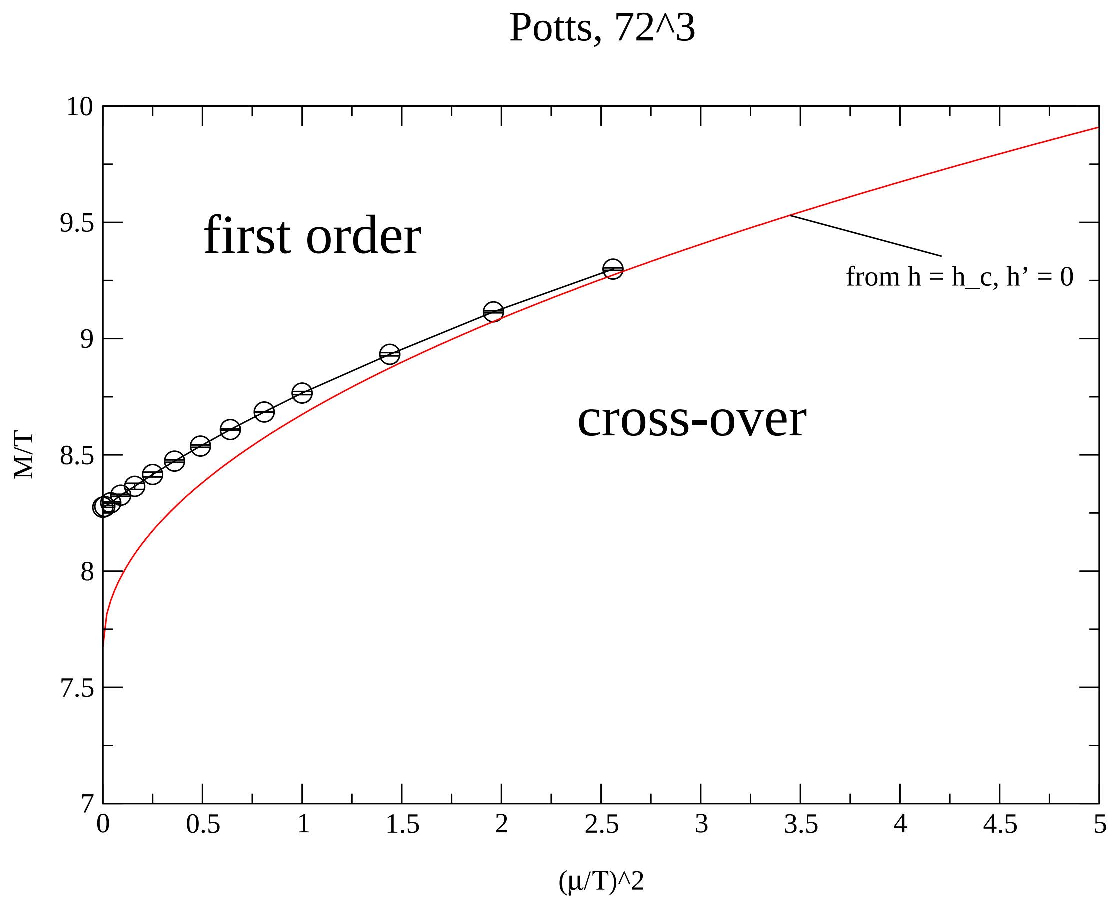
4. Discussion



scenario I for phase diagram in T, μ, m_q,
Ph. de Forcrand and O. Philipsen, hep-lat/0607017



scenario II for phase diagram in T, μ, m_q



HQ QCD - Potts result for phase diagram in T, μ, m_q
S. Kim and Ph. de Forcrand, PoS, Lat2005, 166 (2006), hep-lat/0510069