QCD Critical Point

– what, why, whether, where, and how –

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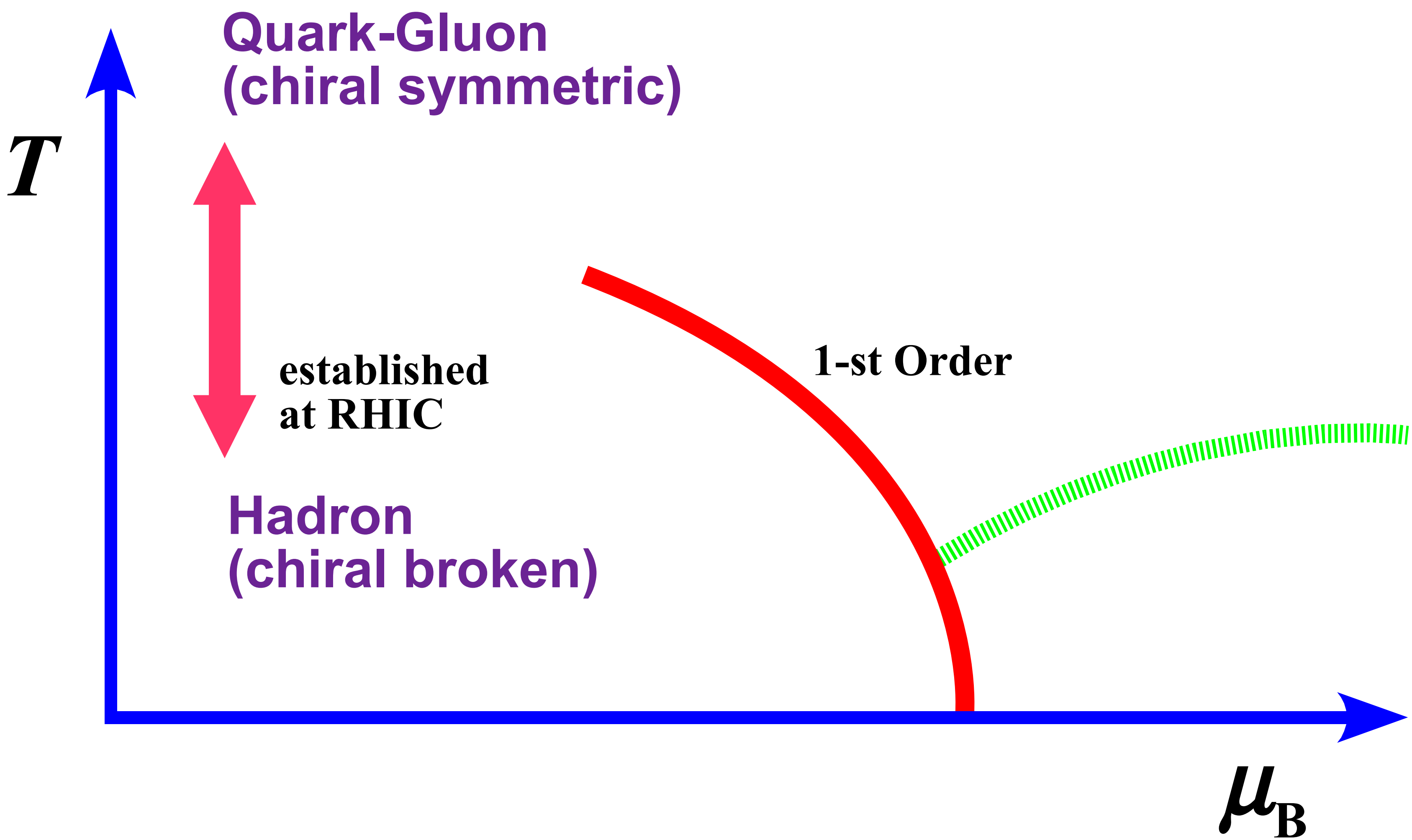
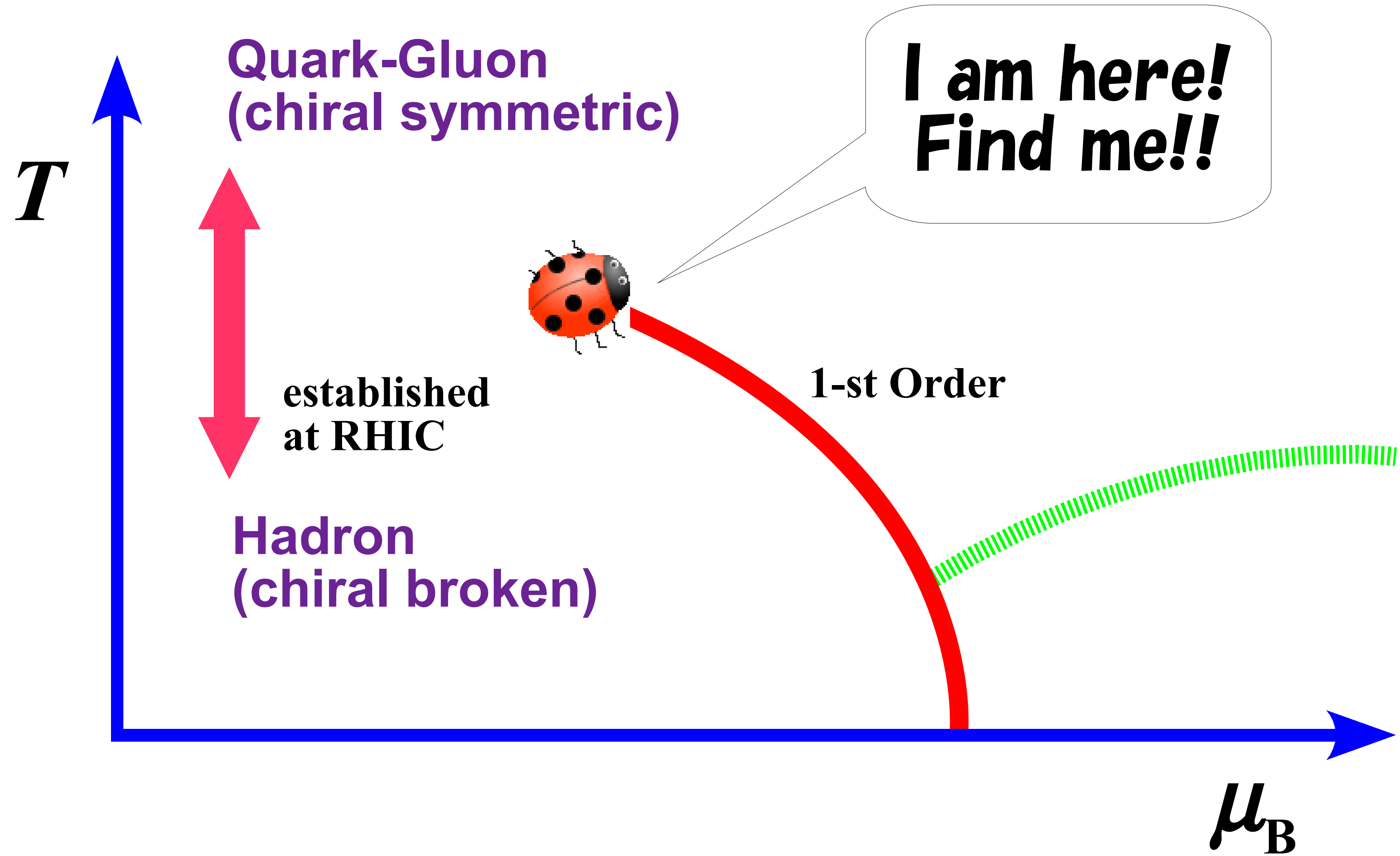
INTRODUCTION

What ?



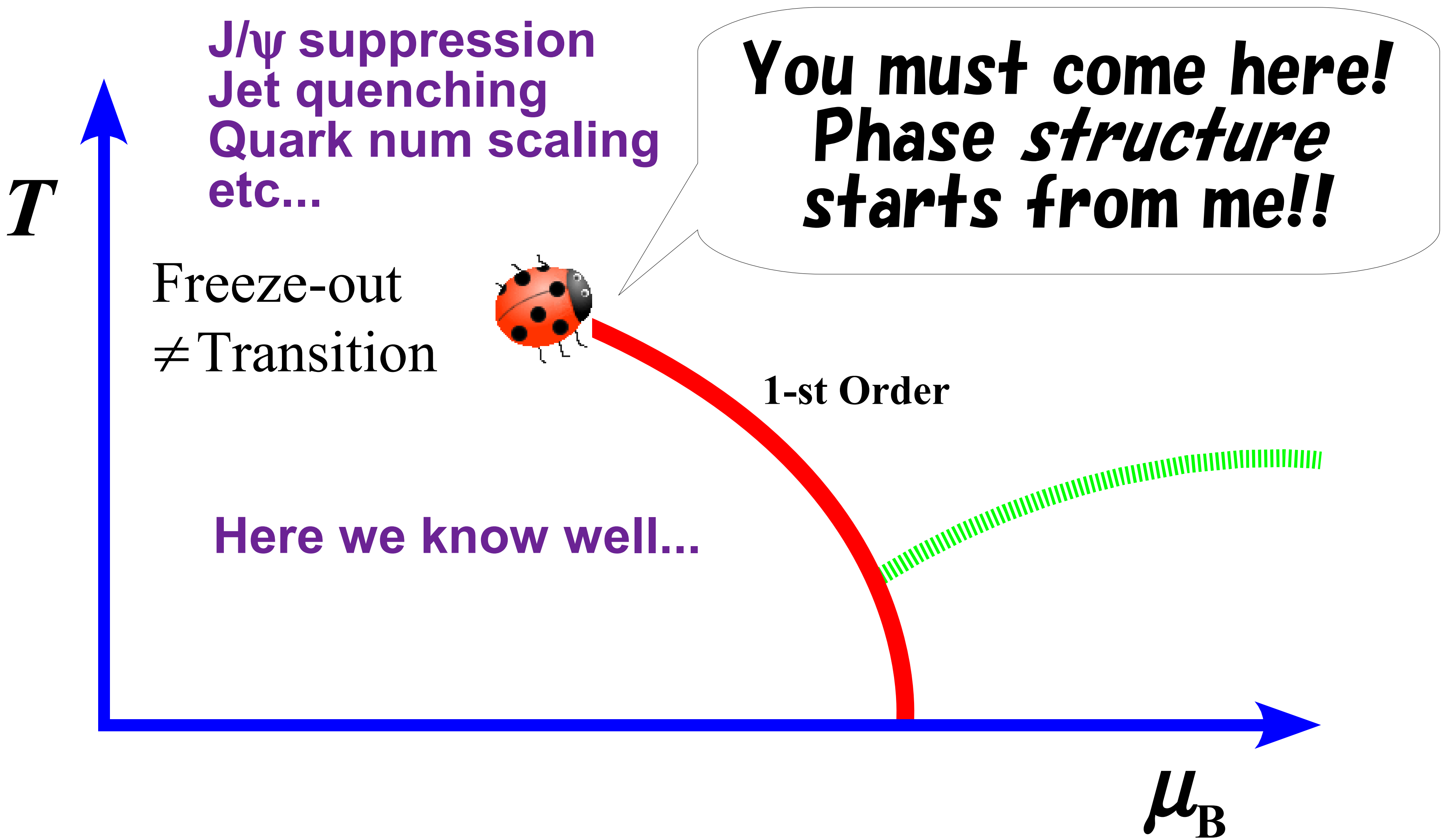
Quark-Gluon
(chiral symmetric)
T
established
at RHIC
1-st Order
Hadron
(chiral broken)
μ_B

What ?



Quark-Gluon
(chiral symmetric)
I am here!
Find me!!
T
established
at RHIC
1-st Order
Hadron
(chiral broken)
μ_B

Why ?


J/ψ suppression
Jet quenching
Quark num scaling
etc...
You must come here!
Phase structure
starts from me!!
T
Freeze-out
≠Transition
1-st Order
Here we know well...
μ_B

Whether, Where, How ???

- **Whether:** the QCD critical point exists?

- **Where:** the QCD critical point lives?

- **How:** the QCD critical point shows itself ?

Which ?

- **Whether:** the QCD critical point exists?
- **Where:** the QCD critical point lives?
- **How:** the QCD critical point shows itself ?
- **Which** is the most relevant question ???

Whether ?

- Affirmative
  - Chiral Effective Models
    - Nambu-Jona-Lasinio model, Linear sigma model
    - Chiral random matrix theory
    - Strong coupling + Large dimensional expansion
  - Lattice QCD Simulations
    - Reweighting method
    - Taylor expansion
    - Canonical ensemble
- Negative
  - Chiral Effective Models
    - Large vector interaction, Small 't Hooft interaction
  - Lattice QCD Simulations
    - Taylor expansion on the critical hypersurface

I am covering…

- Affirmative
  - Chiral Effective Models
    - Nambu-Jona-Lasinio model, Linear sigma model
    - Chiral random matrix theory
    - Strong coupling + Large dimensional expansion
  - Lattice QCD Simulations
    - **Reweighting method**
    - **Taylor expansion**
    - **Canonical ensemble**
- Negative
  - Chiral Effective Models
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  - Lattice QCD Simulations
    - **Taylor expansion on the critical hypersurface**

LATTICE QCD

Difficulty in Lattice QCD

- Fine Lattice
  - Fermion density is saturated at the lattice site density.
  - Low temperature requires large N_t.

- Large Volume
  - 1-st order transition is smeared in a finite-volume box.
  - Canonical ensemble is not equivalent to grand canonical.

- Sign Problem
  - Finite density breaks the validity of computation.
  - Loss of validity confuses with 1-st order transition.

Difficulty in Lattice QCD

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Resource

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Principle!

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Resource

- **Sign Problem**
  - Finite density breaks the validity of computation.
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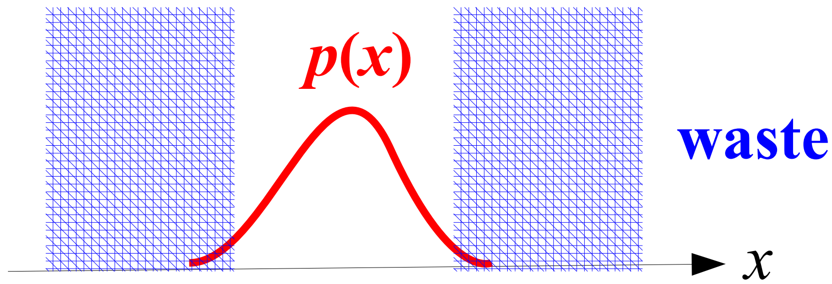
Principle!

Sign Problem

- Monte-Carlo Integration

$$\langle g\rangle = \int_{-\infty}^{\infty} \mathrm{d}x\, p(x) g(x)$$



$$\{x_i\}_p \;\rightarrow\; \langle g\rangle = \sum_i g(x_i) \,/\, \sum_i p(x_i)$$

$$\rightarrow\; \langle g'\rangle = \sum_i g'(x_i) \,/\, \sum_i p(x_i)$$

if g, g' do not affect $\{x_i\}_p$

Lattice QCD $\quad x \rightarrow A, \quad p(x) \rightarrow e^{-S[A]}, \quad g(x) \rightarrow O[A]$

- Sign Problem $\quad p(x) = |p| e^{i\Theta}$ and $|\Theta| > \pi/2$ for some x

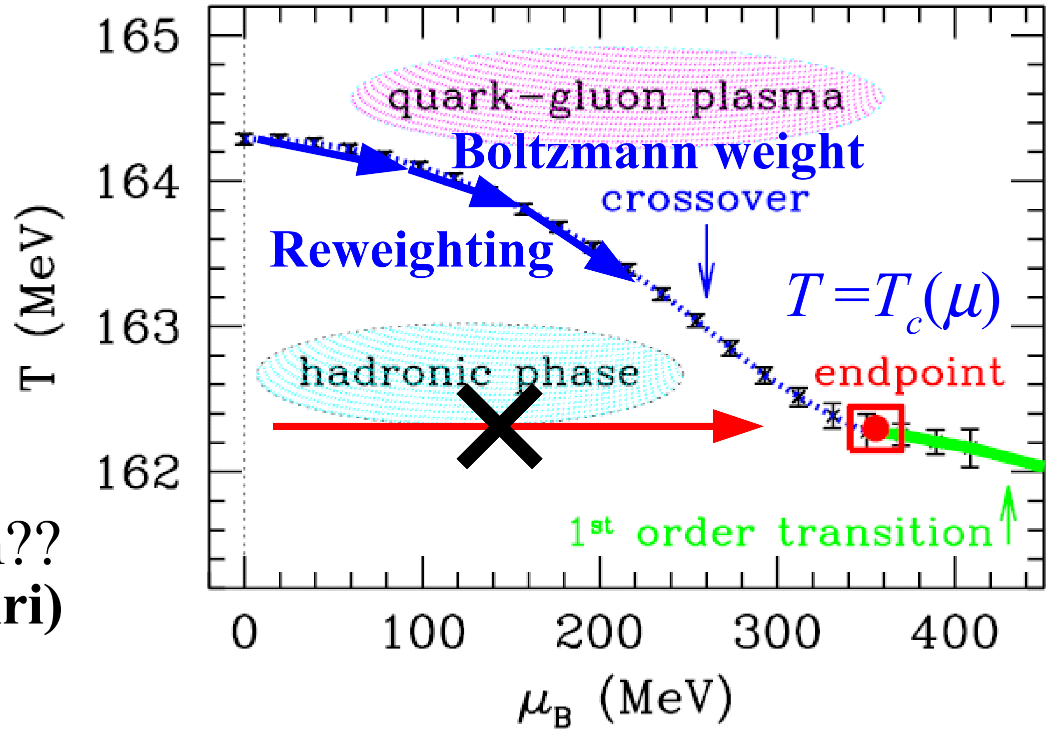
$\{x_i\}_p \rightarrow \{x_i\}_{|p|}$ and $\langle g\rangle = \langle g\, e^{i\Theta}\rangle_{|p|}$ does not work

$p \sim e^{-\# V} \quad e^{i\Theta} \sim e^{-\# V}$

Reweighting

Fodor-Katz *(hep-lat/0402006)*

- Overlap-improving Multi-parameter Reweighting
  - 2+1 flavors, staggered, N_t =4, physical quark mass
  - Idea



- Lee-Yang Zero

Sign Problem
→ Boltzmann weight?
→ 1-st order phase transition??
(S. Ejiri)

- Estimate

$$(T/T_c)_{\rm crit} \simeq 0.99 \quad (\mu_B/T_c)_{\rm crit} \simeq 2.2 \quad T_c = 164\,{\rm MeV}$$

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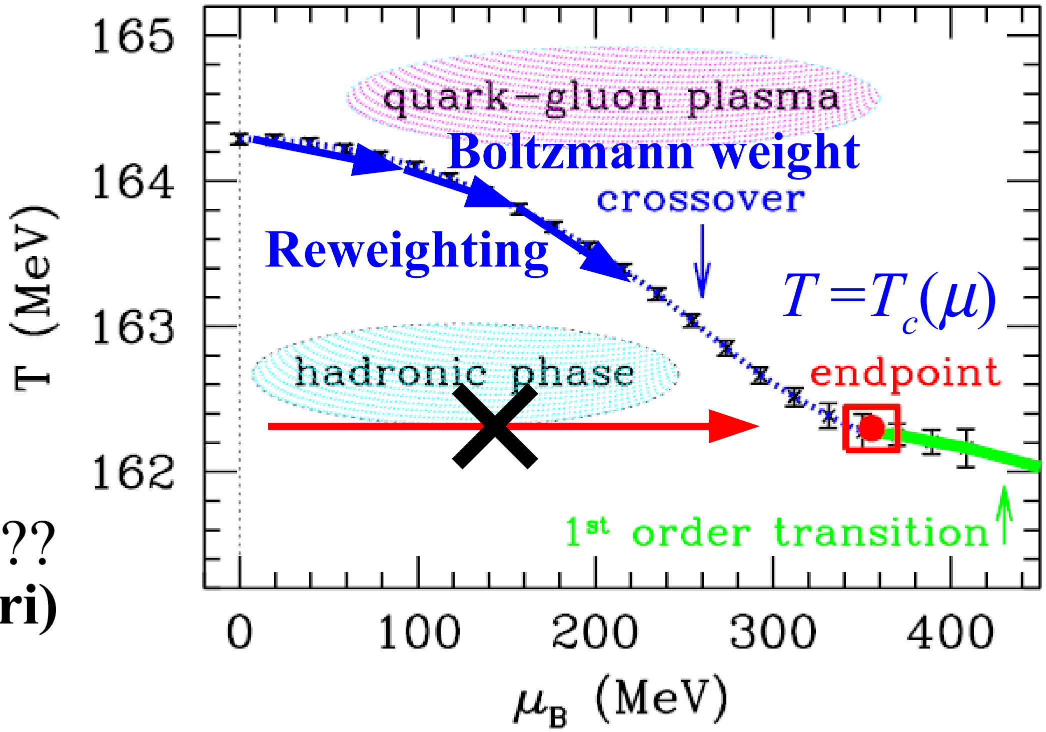


- Lee-Yang Zero

Sign Problem
→ Boltzmann weight?
→ 1-st order phase transition??
(S. Ejiri)

- Estimate

$$(T/T_c)_{\rm crit} < 0.99 \quad (\mu_B/T_c)_{\rm crit} > 2.2 \quad T_c = 164\,{\rm MeV}$$

Gavai-Gupta *(0806.2233 [hep-lat])*

- Taylor Expansion up to Sixth Order
  - 2-flavor, staggered, $N_t = 6$
  - Idea

$$f(\mu) = f(0) + \frac{T^2}{2}\frac{\partial^2 f(0)}{\partial \mu^2}(\mu/T)^2 + \frac{T^4}{24}\frac{\partial^4 f(0)}{\partial \mu^4}(\mu/T)^4 + \cdots$$

- Radius of Convergence **Allton et al.**

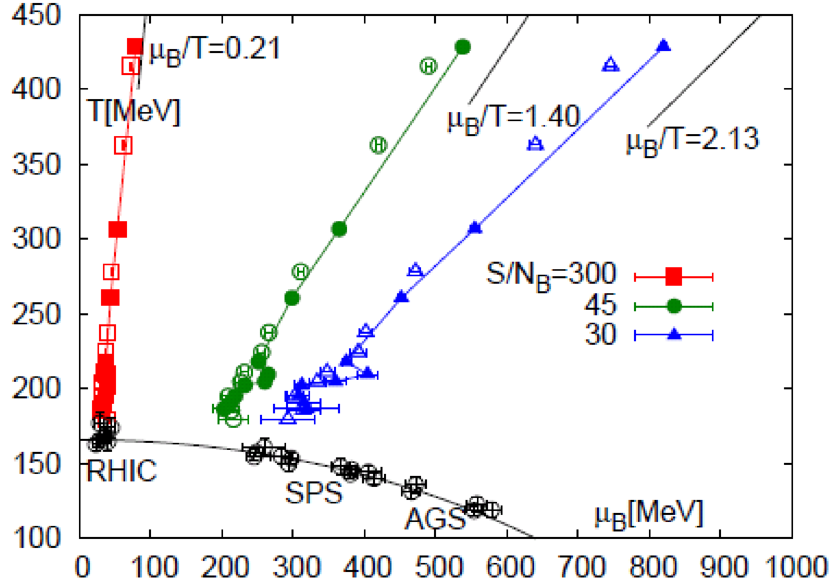
Where the expansion breaks down for $f(x) = \sum c_n x^n$?

c.f. $e^x = \sum \frac{1}{n!} x^n \quad \frac{1}{1-x} = \sum x^n \qquad r = \lim_{n\to\infty} \frac{c_{n-1}}{c_n}$

- Estimate $\quad (T/T_c)_{\rm crit} \simeq 0.94 \quad (\mu_B/T_c)_{\rm crit} \simeq 1.8 \to 1.1\,?$

Schmidt et al. (RBC-Bielefeld) *(0810.0375 [hep-lat])*

- Taylor Expansion up to Sixth Order
  - 2+1 flavor, staggered, N_t =6, physical pion mass
- Radius of Convergence
- Estimate $$(T/T_c)_{\rm crit} \simeq ? \qquad (\mu_B/T_c)_{\rm crit} > 2.7$$



c.f. Isentropic trajectories

Liu et al. (Kentucky) *(0711.2692 [hep-lat])*

- Canonical Ensemble
  - Imaginary μ $\rightarrow$ No sign problem
  - Fourier transform of $\mu_I \rightarrow n_B$

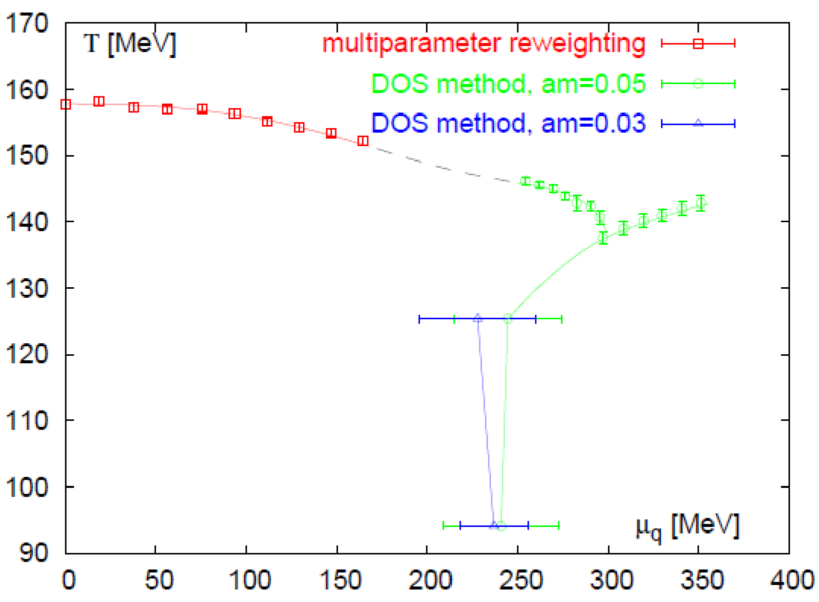
- No conclusive message so far.
  - Difficulty in μ_I integration.
  - Canonical not equivalent with Grand Canonical.

Fodor-Katz-Schmidt *(hep-lat/0701022)*

- Density of States Method $\langle g\rangle=\int_{-\infty}^{\infty}\mathrm{d}x\,p(x)g(x)$

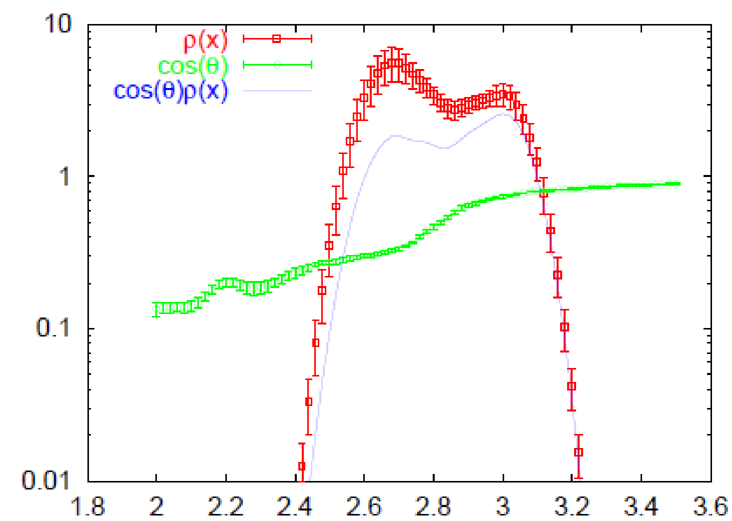
$$\langle g\rangle=\int\mathrm{d}\phi\langle g\rangle_{\phi}\rho(\phi)\ \text{with}\ \rho(\phi)=\int_{-\infty}^{\infty}\mathrm{d}x\,p(x)\delta(\phi-x)$$

$$\langle g\rangle=\int\mathrm{d}\phi\langle g\,e^{i\Theta}\rangle_{\phi}\rho(\phi)\ \text{with}\ \rho(\phi)=\int_{-\infty}^{\infty}\mathrm{d}x|p(x)|\delta(\phi-x)$$



4-flavors

Nishimura et al.



No direct relation to CP

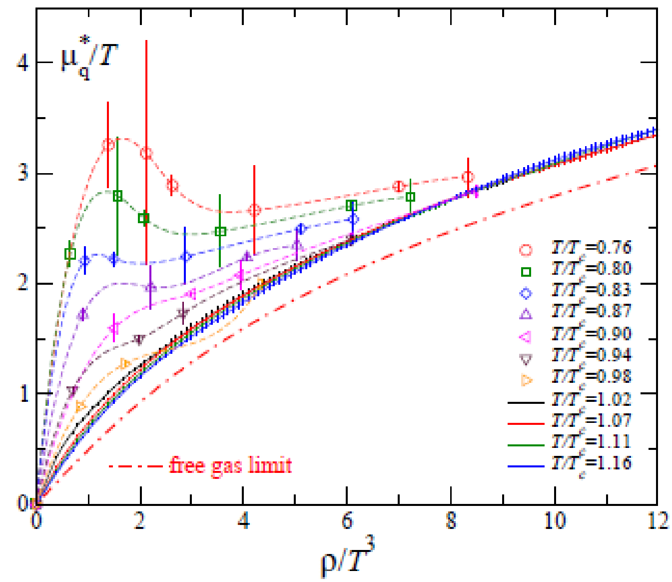
Ejiri (0804.3227 [hep-lat])

- Taylor Expansion + Canonical Ensemble + Density of States Method + Gaussian Approx.
  - 2-flavor, staggered, $N_t = 4$, $m/T = 0.4$
  - Idea $\rho(\phi) \propto e^{-\alpha \Theta^2}$ Fourier transform is done analytically

$$\langle g \rangle = \int \mathrm{d}\phi \langle g e^{i\Theta} \rangle_\phi \rho(\phi) \text{ with } \rho(\phi) = \int_{-\infty}^{\infty} \mathrm{d}x |p(x)| \delta(\phi - x)$$

- Estimate

$$(T/T_c)_{\text{crit}} \simeq 0.76$$
$$(\mu_B/T_c)_{\text{crit}} \simeq 7.5$$



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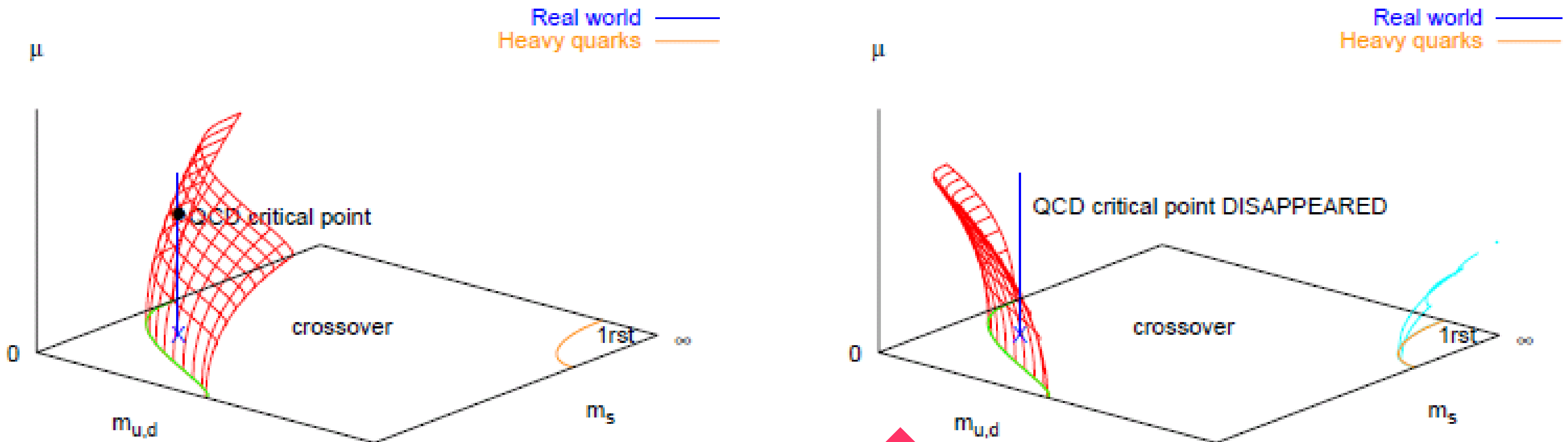
de Forcrand-Philipsen *(0808.1096 [hep-lat])*

- Taylor Expansion up to Fourth Order
  - 3-flavor, staggered, N_t =4, various masses
  - Idea



- Estimate

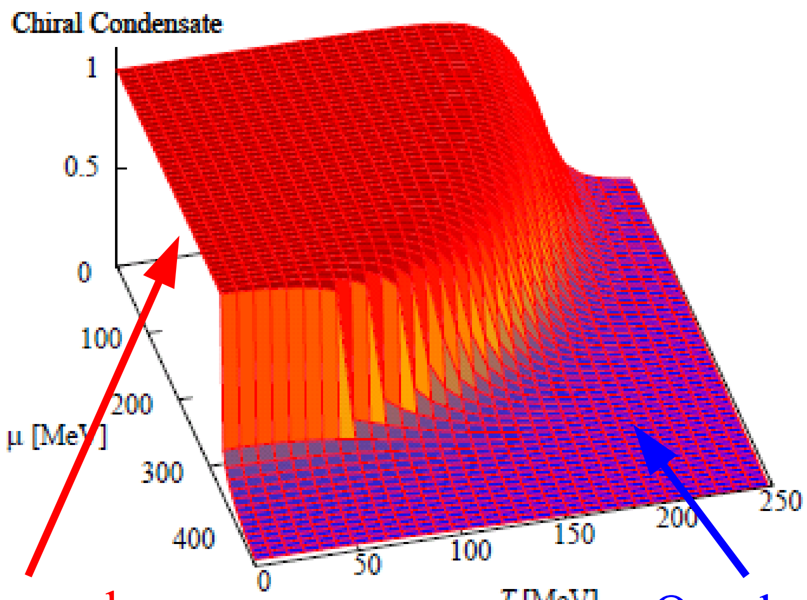
No Critical Point?

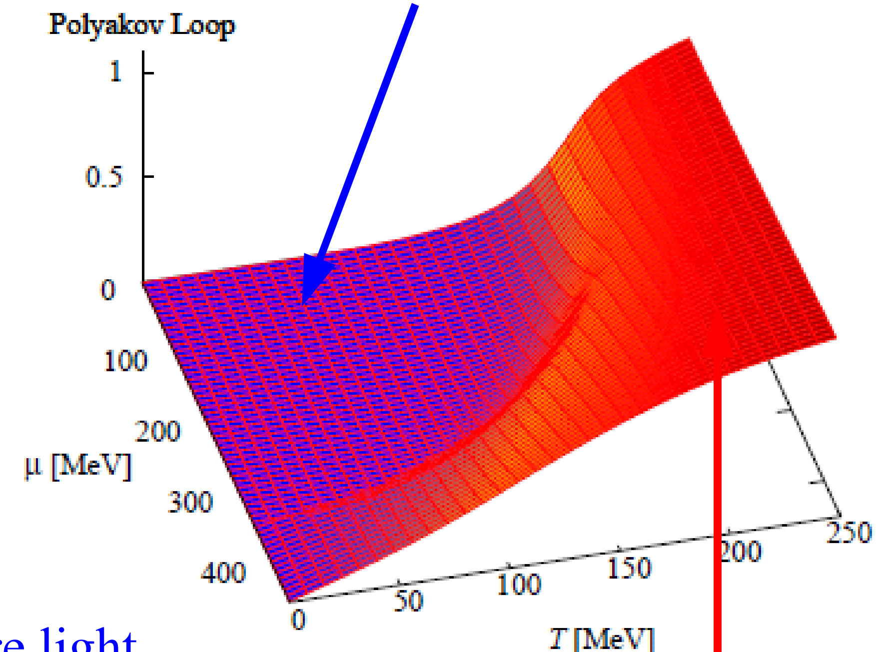
CHIRAL EFFECTIVE MODEL

2+1 Flavor PNJL Model

- 3D Plots – Order Parameters

Quarks cost more energy





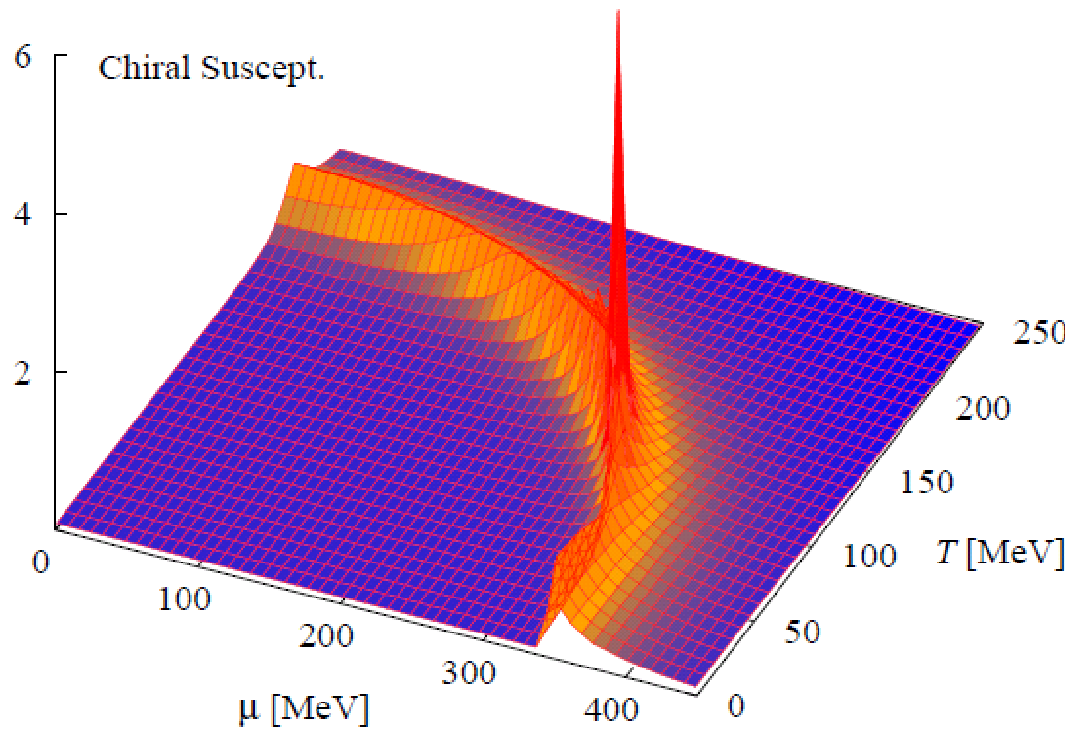
Quarks are heavy

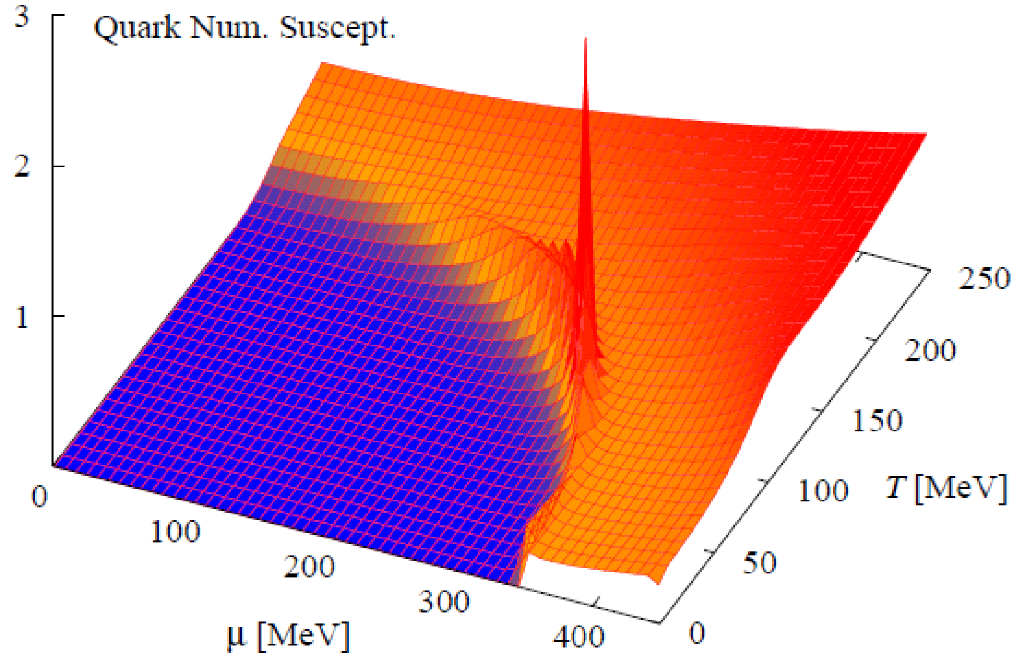
Quarks are light

Quarks cost less energy

PNJL Model

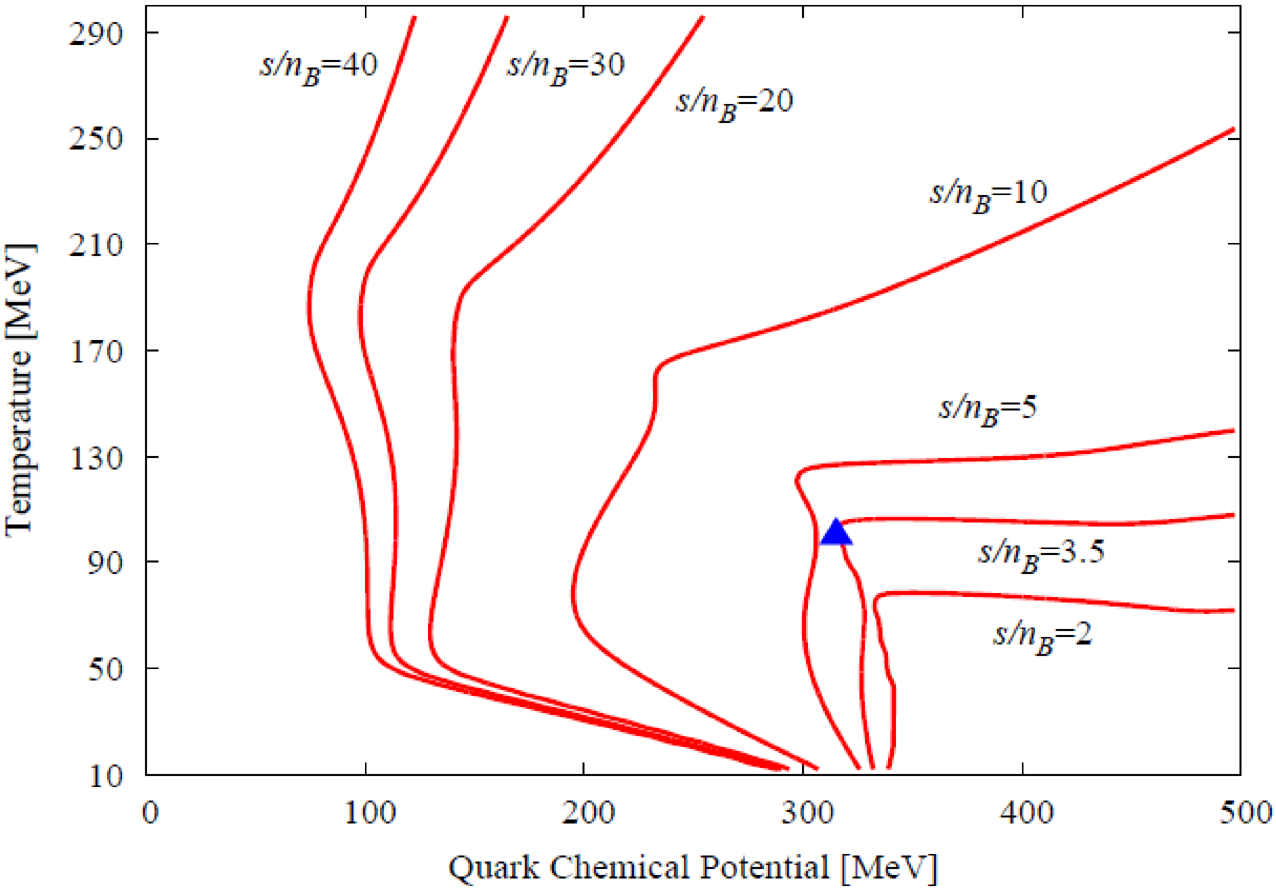
- Chiral and Quark Number Susceptibility

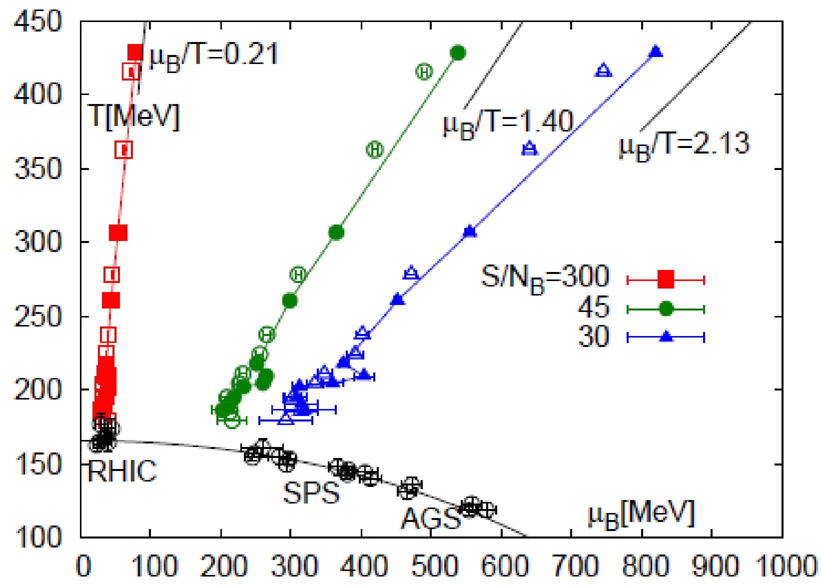




Isentropic Trajectories

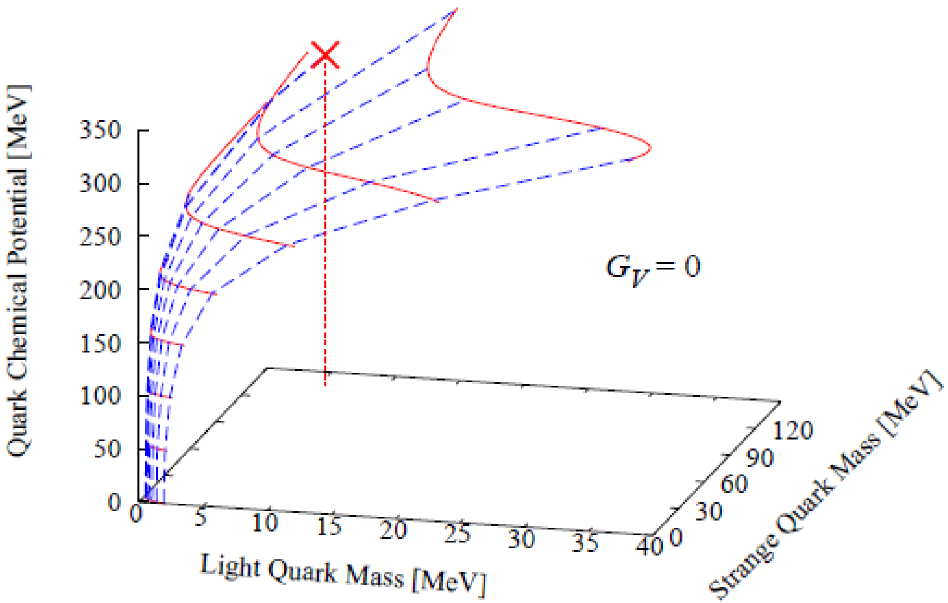
- s/n_B = constant lines

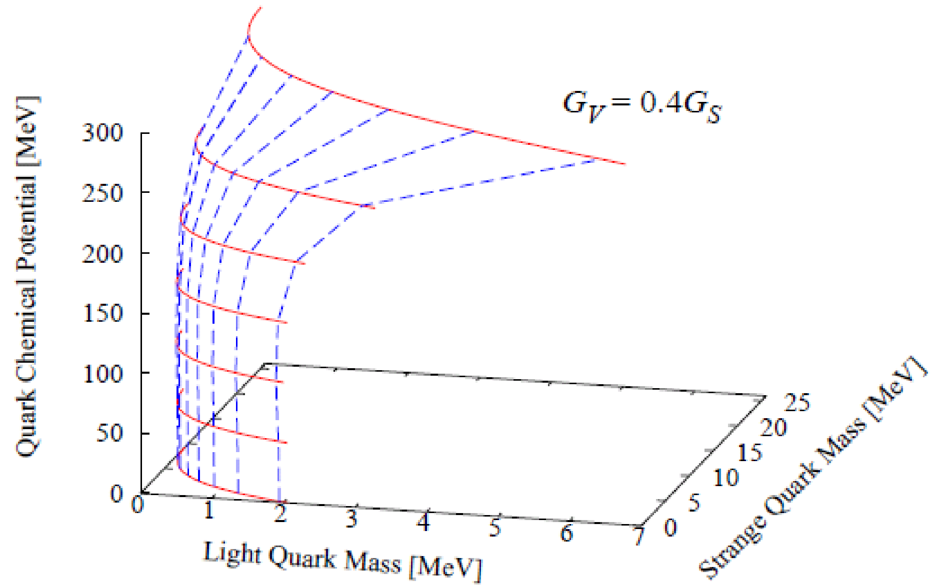


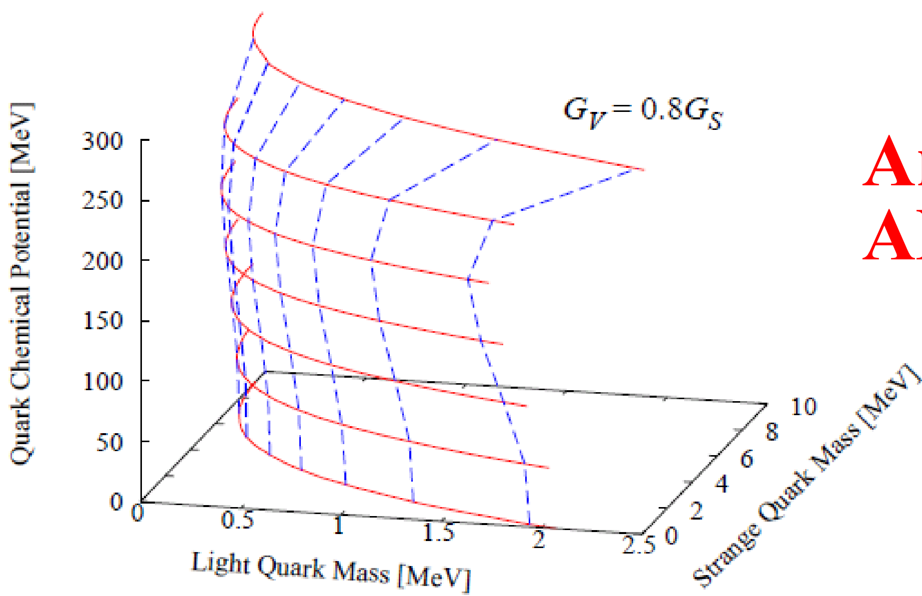


Ambiguities from
't Hooft Interaction
Vector Interaction

Critical Hypersurface



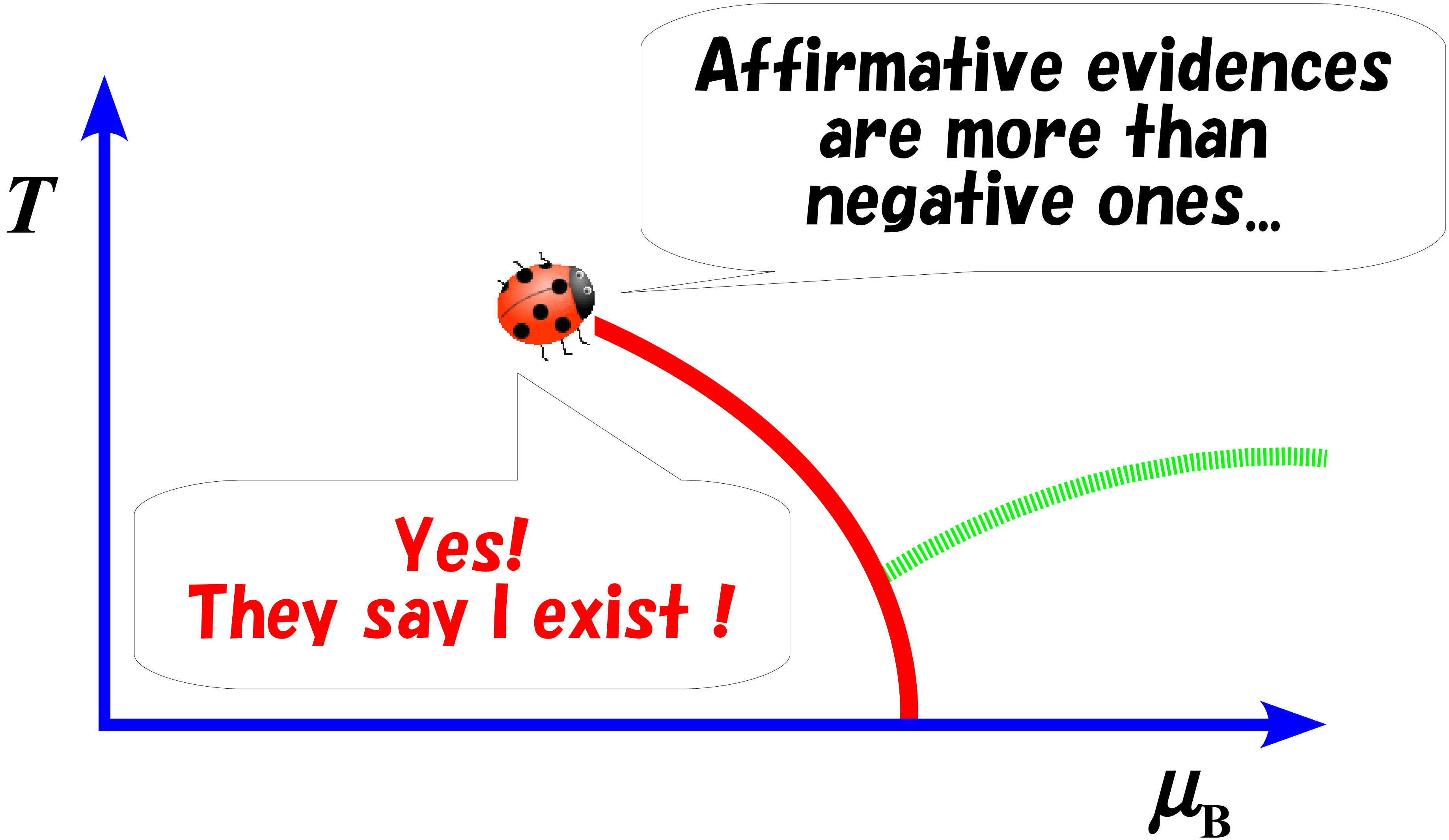




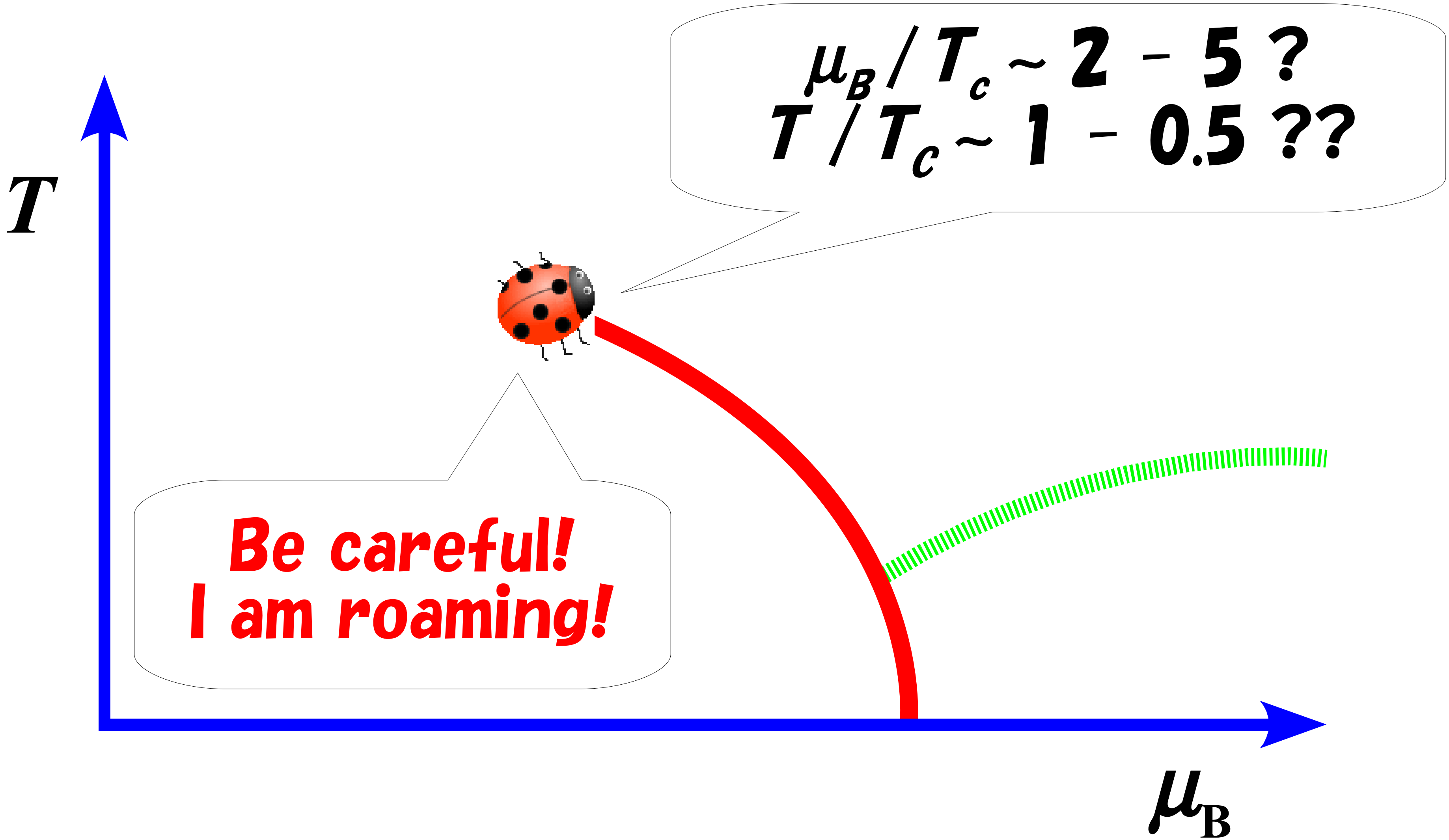
Ambiguity can explain(?) ANYTHING!

CONCLUSIONS

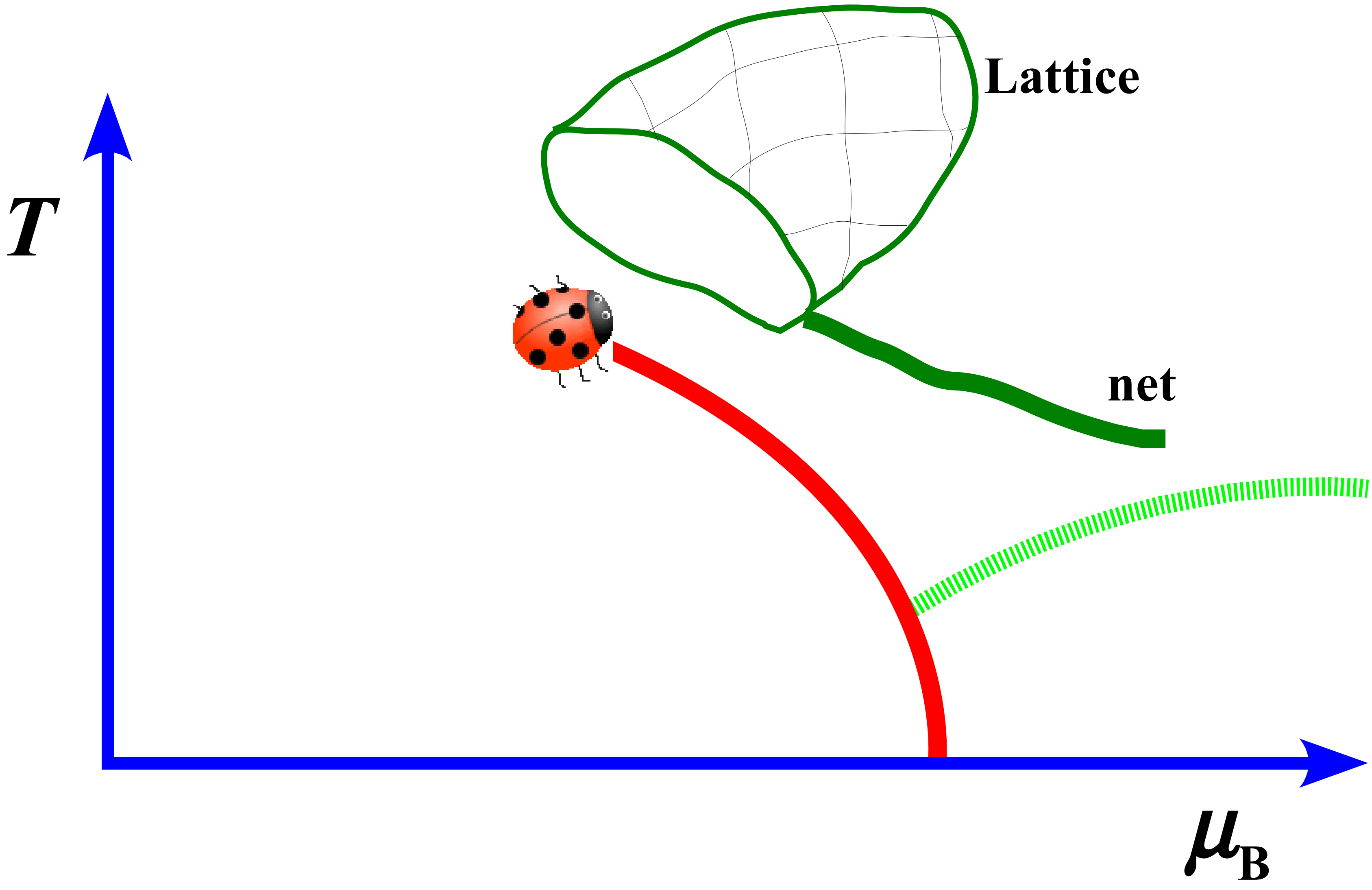
Conclusion – Whether ? –



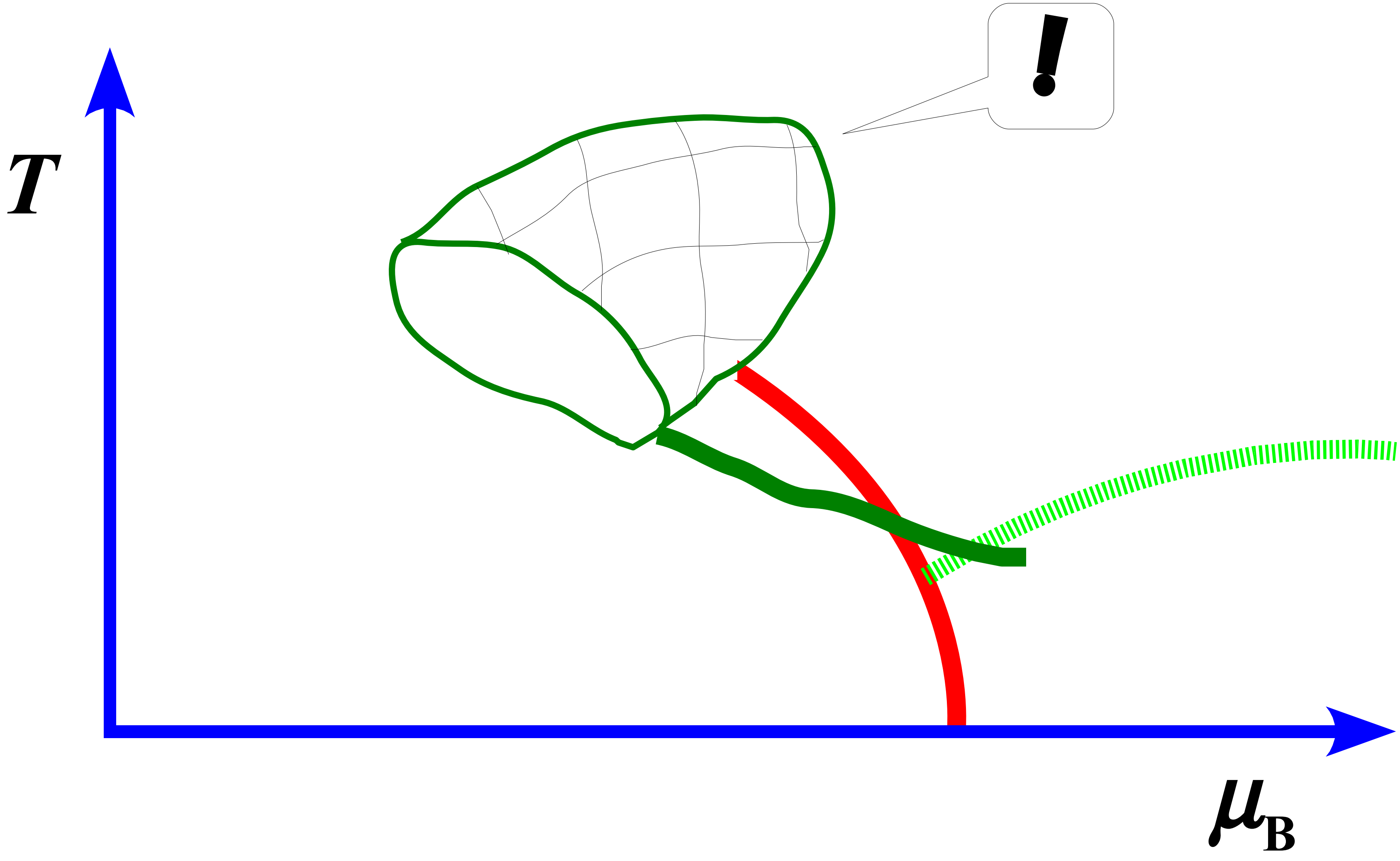
Conclusion – Where ? –



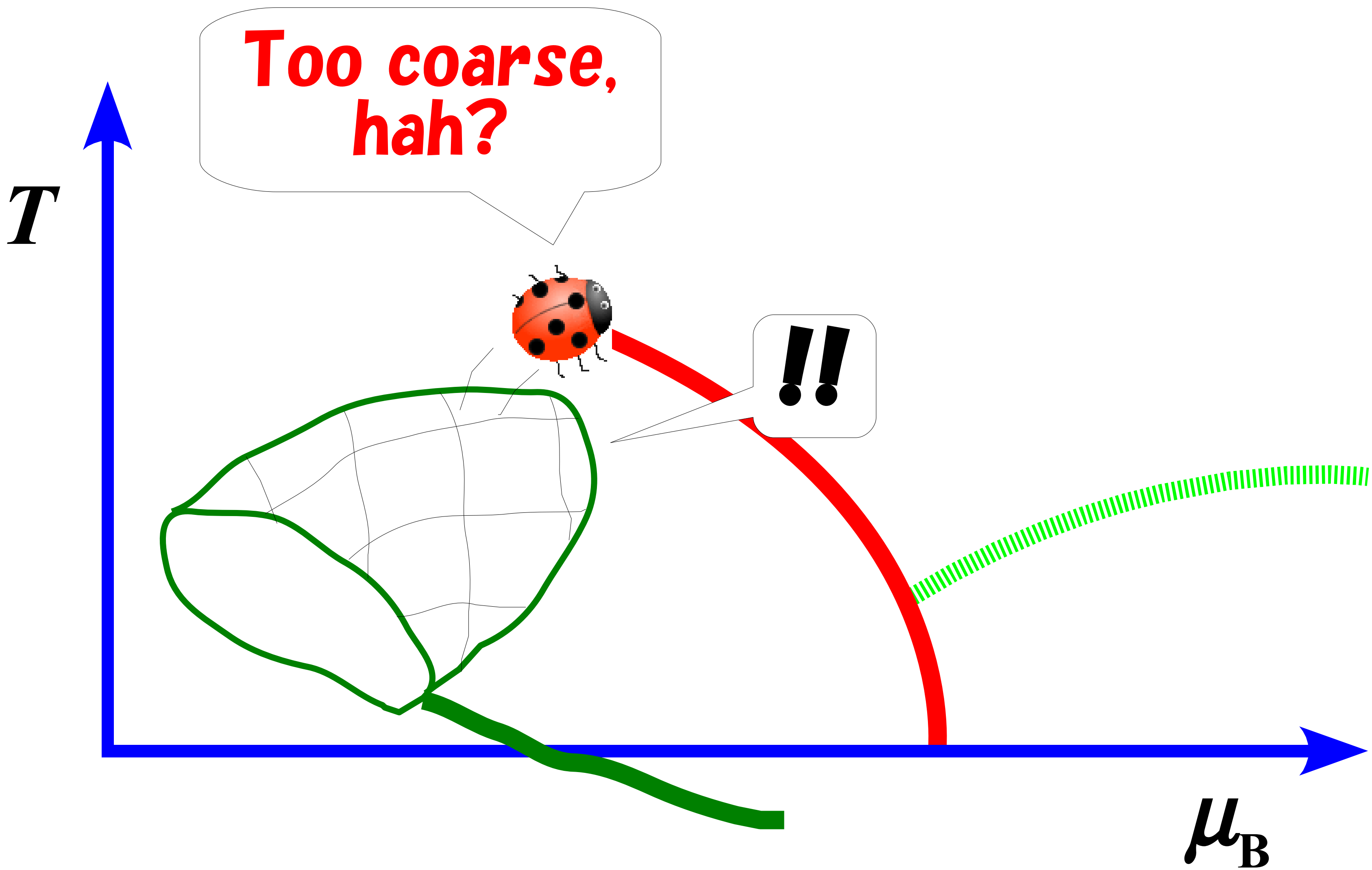
Conclusion – 1 –



Conclusion – 2 –

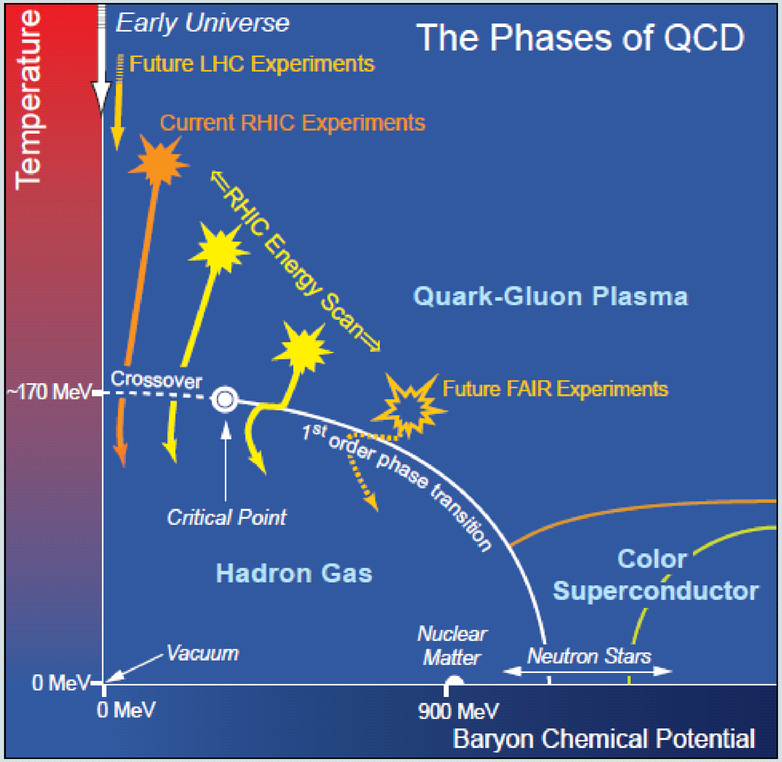


Conclusion – 3 –



To get (draw?) a better net ?

- Experiment – Energy Scan
- Lattice QCD
  - Confirmation of 1-st at zero (small) *T*
  - Matter of time (money)
- Model
  - PNJL is most promising.
  - Ambiguities fixed by Lattice
  - Color superconductivity



Schematic QCD phase diagram for nuclear matter. The solid lines show the phase boundaries for the indicated phases. The solid circle depicts the critical point. Possible trajectories for systems created in the QGP phase at different accelerator facilities are also shown.