Large Density Phase of FT Two-Color QCD

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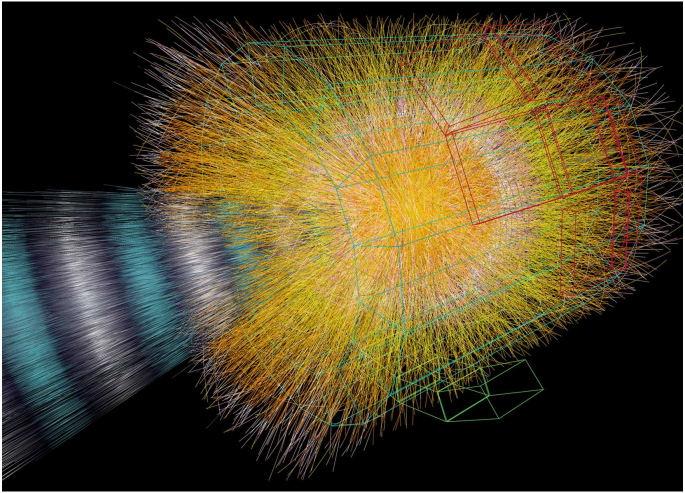
Sejong University

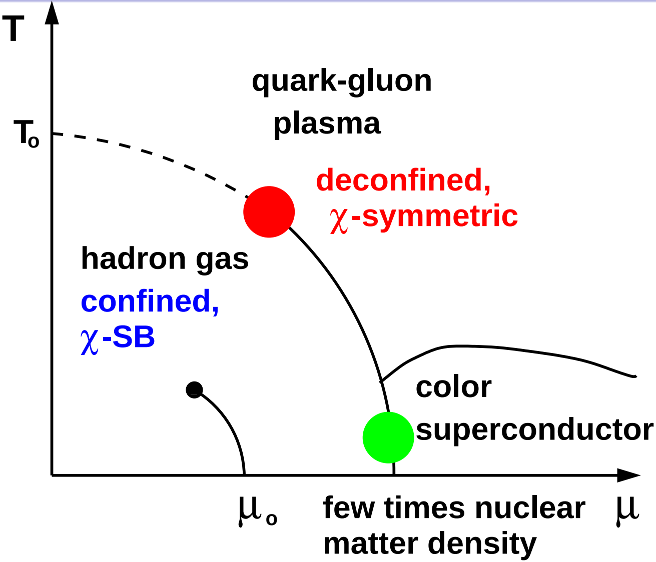
Outline

Motivation

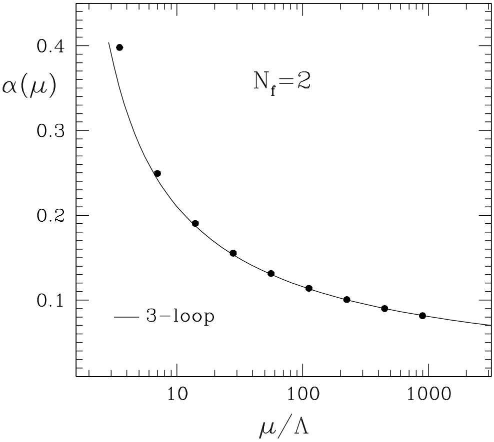


from http://doc.cern.ch/archive/electronic/cern/others/PHO/photo-bul/bul-pho-2007-073.jpg

Motivation


T
T_o
quark-gluon
plasma
deconfined,
χ-symmetric
hadron gas
confined,
χ-SB
color
superconductor
μ_o
few times nuclear
matter density
μ

Motivation



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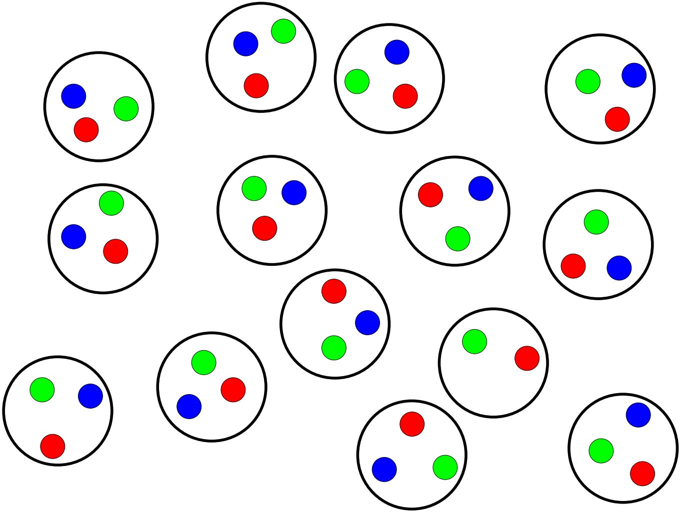
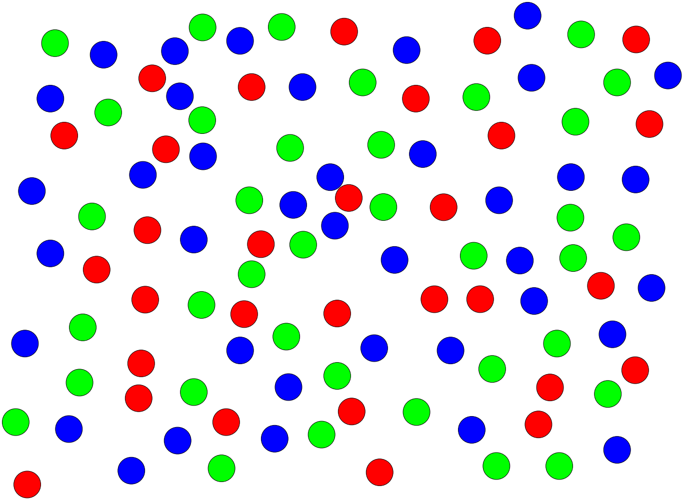


Motivation



Finite Temperature/Density Lattice Field Theory

- statistical mechanics
- consider Ising model

$$Z = \mathrm{Tr} e^{-H} \tag{1}$$

where

$$H = -J \sum_x \sigma_x \sigma_{x+i} - h \sum_x \sigma_x \tag{2}$$

with $\sigma_x = \pm 1$

Finite Temperature/Density Lattice Field Theory

- symmetry $\longrightarrow$ order parameter

$$\langle M\rangle = \langle\sum_x \sigma_x\rangle = \frac{\mathrm{Tr}\sum_x \sigma_x e^{-H}}{\mathrm{Tr} e^{-H}} \quad (1)$$

- non-symmetry related quantity

$$\langle E\rangle = \langle H\rangle = \frac{\mathrm{Tr} H e^{-H}}{\mathrm{Tr} e^{-H}} \quad (2)$$

Finite Temperature/Density Lattice Field Theory

- $k_B = 1$

$$Z_C = \mathrm{Tr} e^{-\frac{H}{T}} = \sum_{\phi} \langle \phi | e^{-\frac{H}{T}} | \phi \rangle \quad (1)$$

Finite Temperature/Density Lattice Field Theory

- recall

$$\begin{aligned} F(q_f, q_i) &= \langle q_f | e^{-iHt} | q_i \rangle & (1) \\ &= \int dq_1 dq_2 \cdots dq_{N-1} \langle q_f | \hat{T} | q_{N-1} \rangle q_{N-1} | \hat{T} | q_{N-2} \rangle \cdots \langle q_1 | \hat{T} | q_i \rangle \\ &\to \int \mathcal{D}x(t) e^{i \int dt\, L} \\ &\to \int \mathcal{D}x(\tau) e^{-\int d\tau\, L_E} & (2) \end{aligned}$$

under $t \to i\tau$

Finite Temperature/Density Lattice Field Theory

- bosonic field: periodic in the time direction

- fermionic field: anti-periodic in the time direction

$$Z_C = \sum_{\phi} \langle \phi | e^{-S_E} | \phi \rangle \tag{1}$$

where

$$S_E = \int d^3x \int_0^{1/T} L_E \tag{2}$$

Finite Temperature/Density Lattice Field Theory

$$S_E = -\frac{1}{4}\mathrm{Tr}\log(M^\dagger M) + \sum_{x,\mu\nu}\frac{1}{6g^2}\mathrm{Tr}[1 - P_{\mu\nu}(x)] \qquad (1)$$

where

$$M_{x,y} = \frac{1}{2}\sum_{x,\mu}\overline{\chi}(x)\eta_\mu(x)\left[U_\mu(x)\chi(x+\hat{\mu}) - U^\dagger_\mu(x-\hat{\mu})\chi(x-\hat{\mu})\right] \qquad (2)$$

$\eta_\mu(x)$ is the Kawamoto-Smit phase factor in the staggered fermion method

Finite Temperature/Density Lattice Field Theory

- symmetry $\longrightarrow$ order parameter
- chiral symmetry

$$\langle \overline{\psi} \psi \rangle \tag{1}$$

- $Z(3)$ symmetry for quenched theory

$$\langle \prod_t U_t(x,t) \rangle \tag{2}$$

Finite Temperature/Density Lattice Field Theory

$$Z_{GC} = \mathrm{Tr} e^{-\frac{H-\mu N}{T}} \tag{1}$$

where $N = \overline{\psi}\gamma_4\psi$

- F. Karsch and P. Hasenfratz, PLB125, 308(1983)

$$M_{x,y} = \frac{1}{2}\sum_{x,\mu}\overline{\chi}(x)\eta_\mu(x)\left[U_\mu(x)e^{\mu}\chi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})e^{-\mu}\chi(x-\hat{\mu})\right] \tag{2}$$

Finite Temperature/Density Lattice Field Theory

- complex action problem

  $\longrightarrow$ model study

2-color gauge theory

• Two color QCD or SU(2) gauge theory in large chemical potential is different from QCD

• For SU(2),

$$(C\gamma_5)\tau_2 M(\mu)(C\gamma_5)^{-1}\tau_2 = M^*(\mu) \quad (1)$$

• $\det M(\mu)$ is real (but does not mean that it is positive)

• There is spontaneous chiral symmetry breaking
$\longrightarrow$ pion is light

• But qq is a color singlet $\longrightarrow$ diquark condensate does not break color symmetry

2-color gauge theory

- Model for QCD but
- gluon sector is similar to QCD
- large chemical potential region can be studied

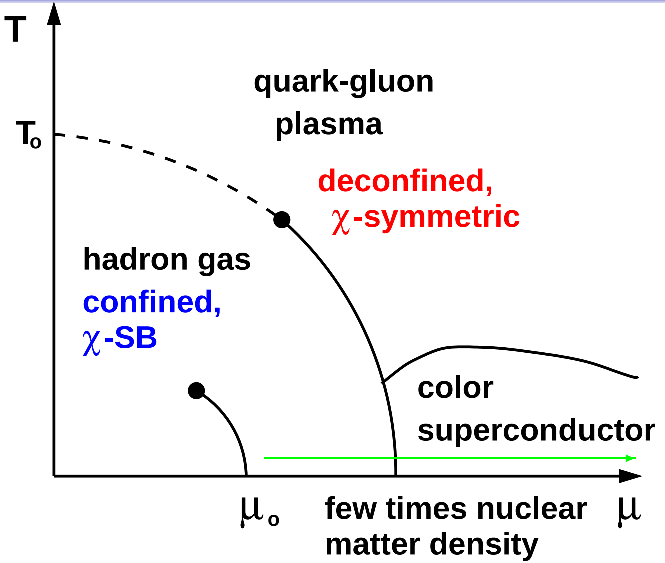
2-color gauge theory

- BEC phase : Kogut et al, Nucl. Phys. B582 (2000)477
- $n_q \propto f_\pi^2(\mu - \mu_0)$
- $\langle qq \rangle \propto \sqrt{1 - (\frac{\mu_0}{\mu})^4}$

2-color gauge theory

- BCS phase
- $n_q \propto \mu^3$
- $\varepsilon_F \propto \mu^4$
- $\langle qq \rangle \propto \Delta\mu^2$

Low T, μ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Low T, μ

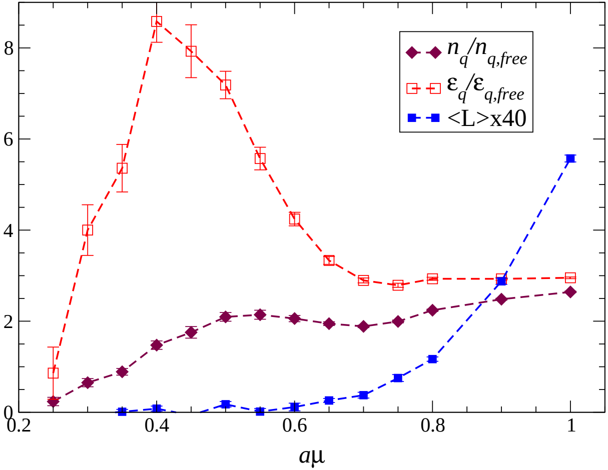
- Two color QCD with heavy quark:

  S. Hands, S. K., J.-I. Skullerud, Eur.Phys.J.C48:193,2006

  $8^3 \times 16, \beta = 1.7$ Wilson quark

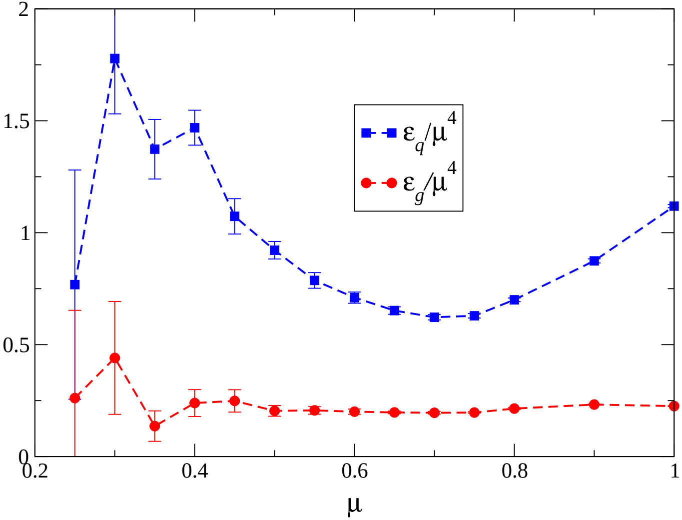
  $\kappa = 0.168, m_\pi a \sim 0.8$

Low T, μ

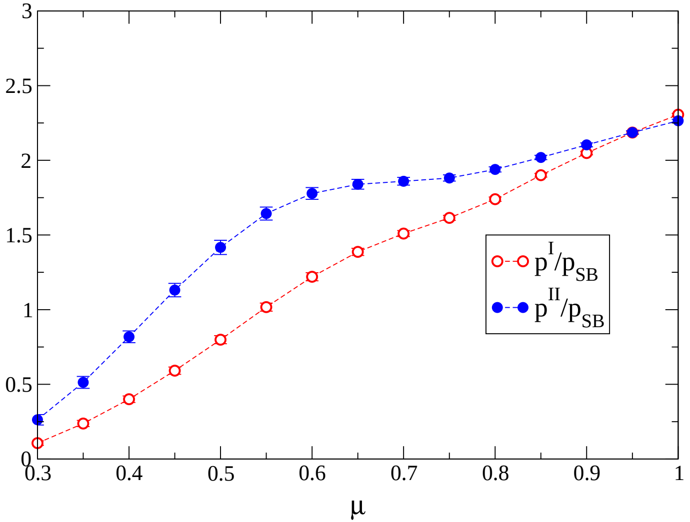


$n_q/n_{q,free}$
$\varepsilon_q/\varepsilon_{q,free}$
<L>x40
0
2
4
6
8
0.2
0.4
0.6
0.8
1
$a\mu$

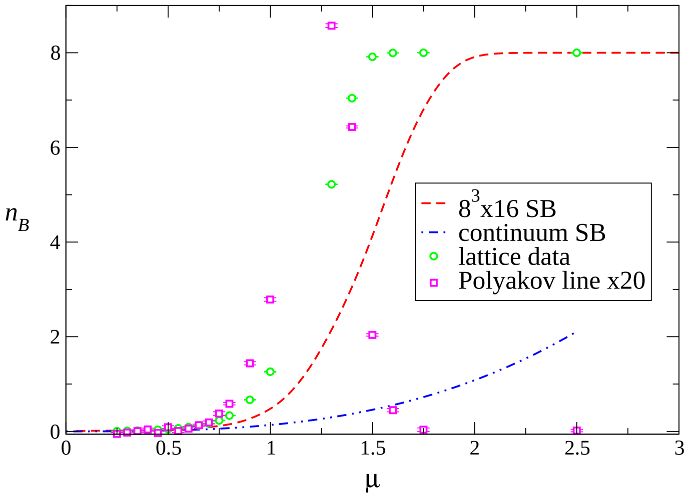
Low T, μ



Low T, μ



Low T, μ



n_B
μ
0
2
4
6
8
0
0.5
1
1.5
2
2.5
3
8^3x16 SB
continuum SB
lattice data
Polyakov line x20

Low T, μ

- Hint of three different phases

  Hadronic phase

  Bose-Einstein Condensed(BEC) phase

  Bardeen-Cooper-Schrieffer(BCS) phase

- BEC-BCS transition is not a sharp transition

  strong attraction $\longrightarrow$ tightly bound boson

  $\longrightarrow$ boson condesation

  weaker attraction $\longrightarrow$ loosely bound Cooper pair

  $\longrightarrow$ superconducting phase

Low T, μ

- P. Nozieres, S.Schmitt-Rink, J. of Low Temp. Phys. 59, 195(1985)

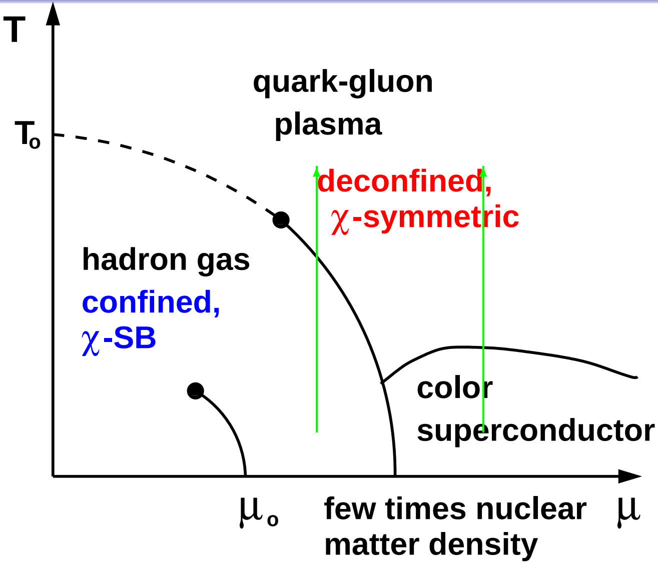
  finite temperature transition of BEC phase is different

  $$T_c = (2\pi/M)(N_p/2.612)^{2/3}$$

  from that of BCS phase

  $$T_c = (e^{\gamma}/\pi)\Delta_F$$

Finite T, μ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Finite T, μ

• temperature is defined as $T = \frac{1}{N_\tau a}$

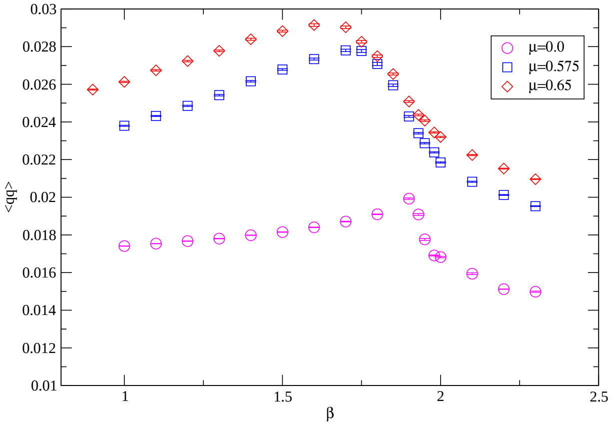
$\leftarrow$ a is controlled by gauge coupling constant
$\leftarrow$ different temperature means a different gauge coupling constant

• periodic boundary condition on bosonic field, anti-periodic boundary condition on fermionic field

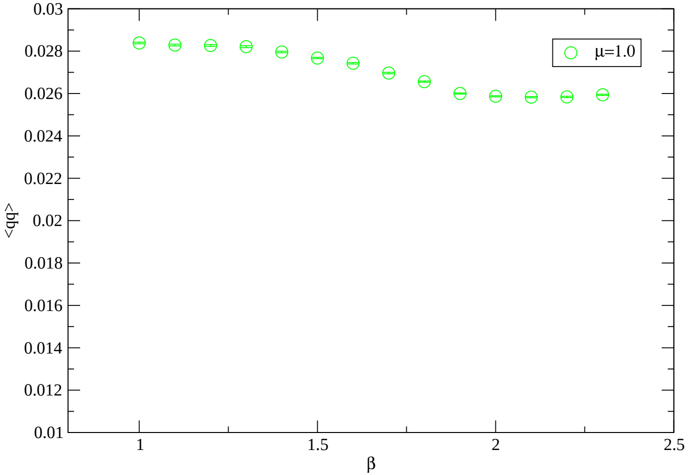
• no. of spatial lattice sites (N_s) should be bigger than N_τ

• phases are distinguished by order parameters

Finite T, μ


0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
<qq>
1
1.5
2
2.5
β
μ=0.0
μ=0.575
μ=0.65

Finite T, μ


μ=1.0
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0.03
0.028
0.026
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0.022
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0.018
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0.014
0.012
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1
1.5
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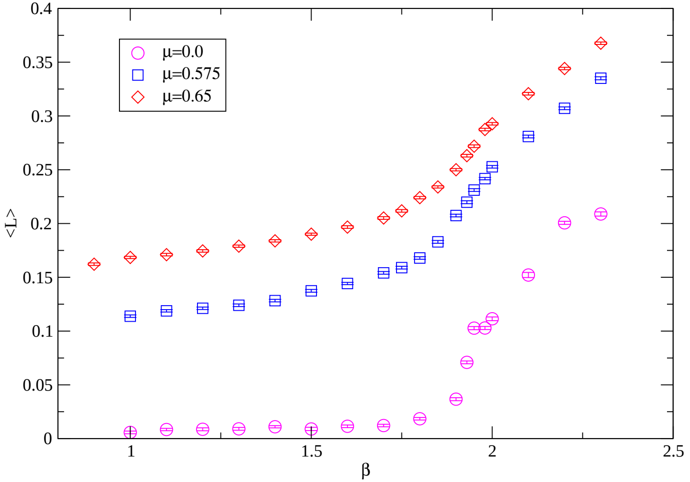
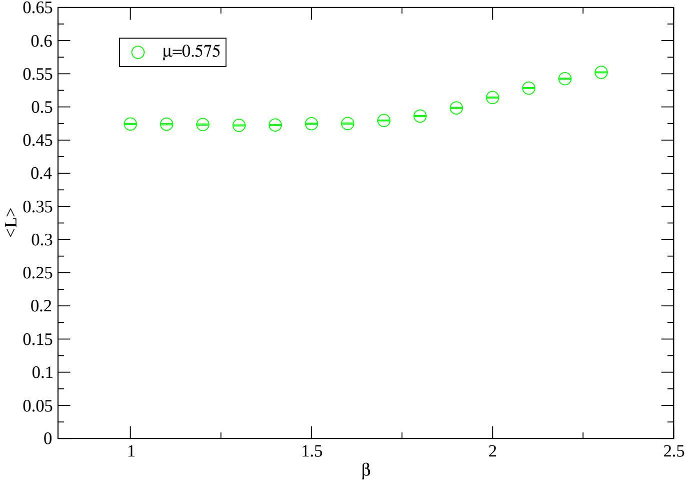
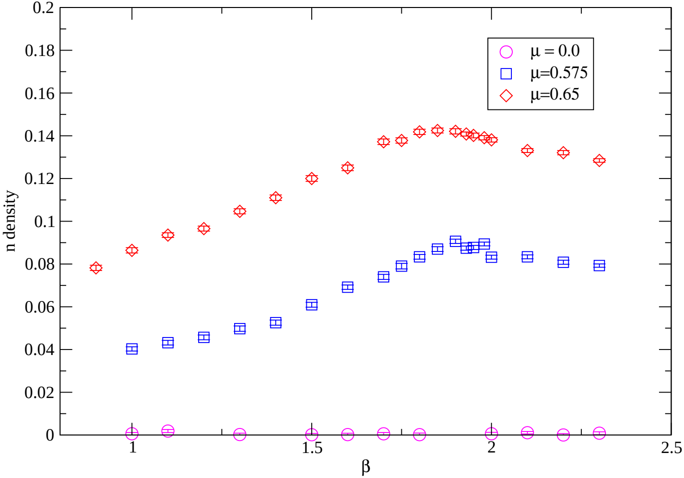
Finite T, μ



Finite T, μ

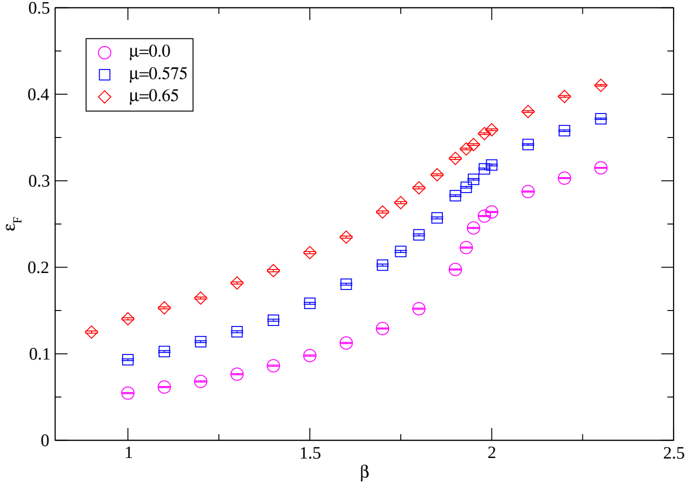


Finite T, μ

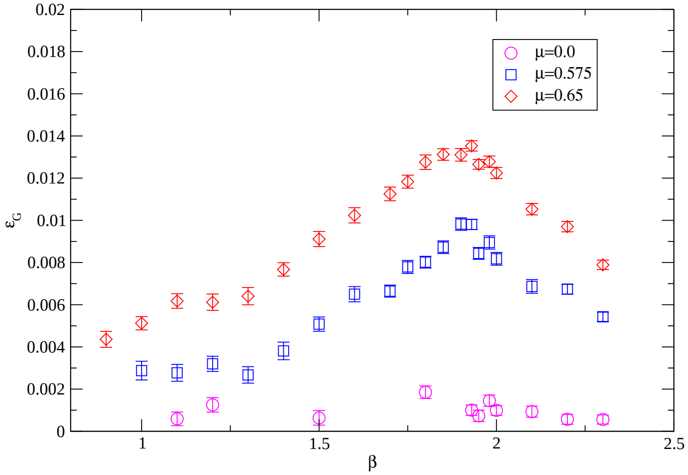


0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
n density
1
1.5
2
2.5
β
μ = 0.0
μ=0.575
μ=0.65

Finite T, μ



Finite T, μ



0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
ε_G
1
1.5
2
2.5
β
μ=0.0
μ=0.575
μ=0.65

Discussion

- diquark condensate behavior shows interesting behavior

  for $\mu = 0.575, 0.65$, the diquark condensate doesn't change much

  for $1.75 < \beta < 2.0$

- further study is needed

- many parameters to scan $\rightarrow$ Grid computing

Discussion