Quark number susceptibility in hQCD: revisited

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(APCTP_JRG/POSTECH)

Plan

- Why qSUS?
- Holographic QCD:
  (a) Hard wall model
  (b) Retarded Green's function
- qSUS with finite chemical potential
- Discussion

Why qSUS?

Quark number SUS as a chiral symmetry order parameter

$$r = \rho_B^{CS}/\rho_B^{CB}$$

In an ideal QGP

$$\rho_B^{CS} = \frac{4}{\pi^2}\mu T^2$$

In the hadron gas phase

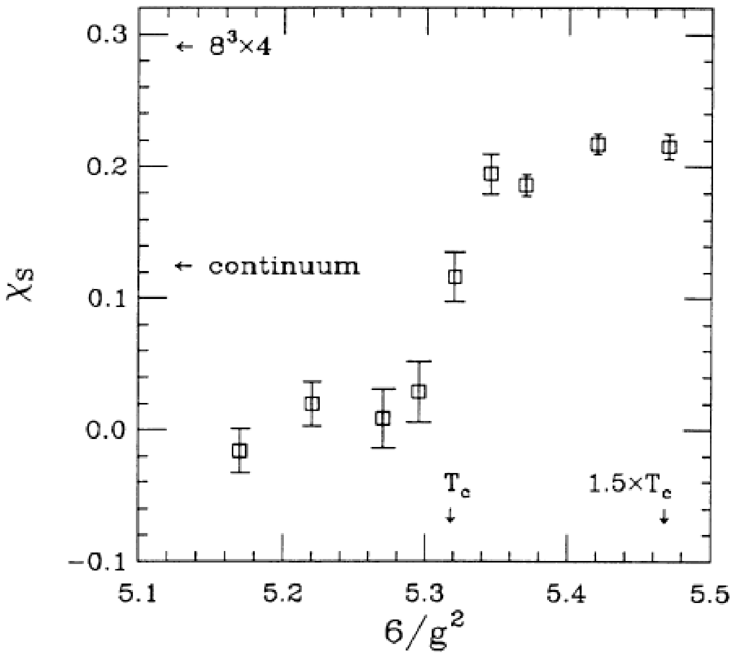
$$\rho_B^{CB} = \frac{1}{2\pi^{3/2}}(2mT)^{3/2}\frac{\mu}{T}e^{-\beta m} .$$

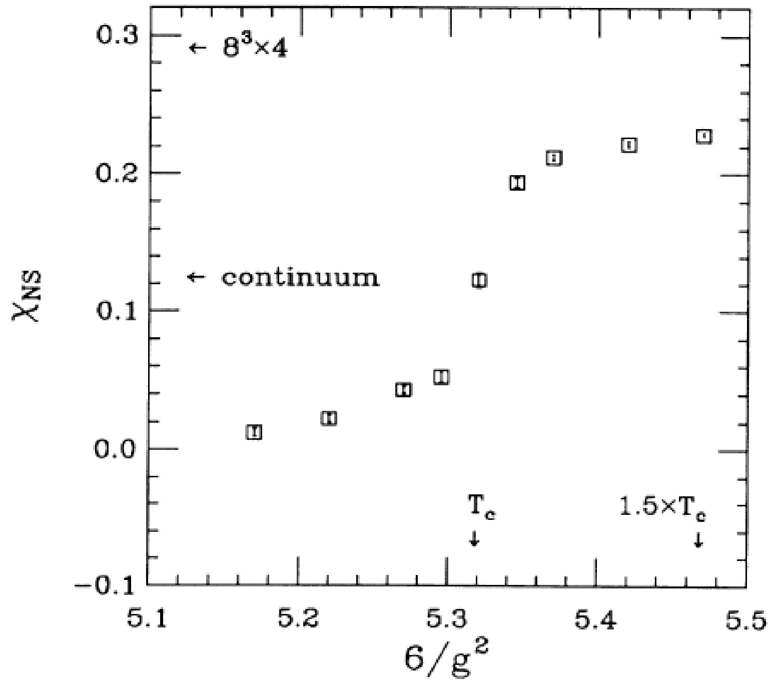
For example, if we take $T \sim 150$ MeV, and $m \sim 1$ GeV, then $r \sim 10^2$.

$$\rho_B = \mu\left(\frac{\partial}{\partial\mu}\rho_B\right)\Big|_{\mu=0} \equiv \mu\kappa_B$$

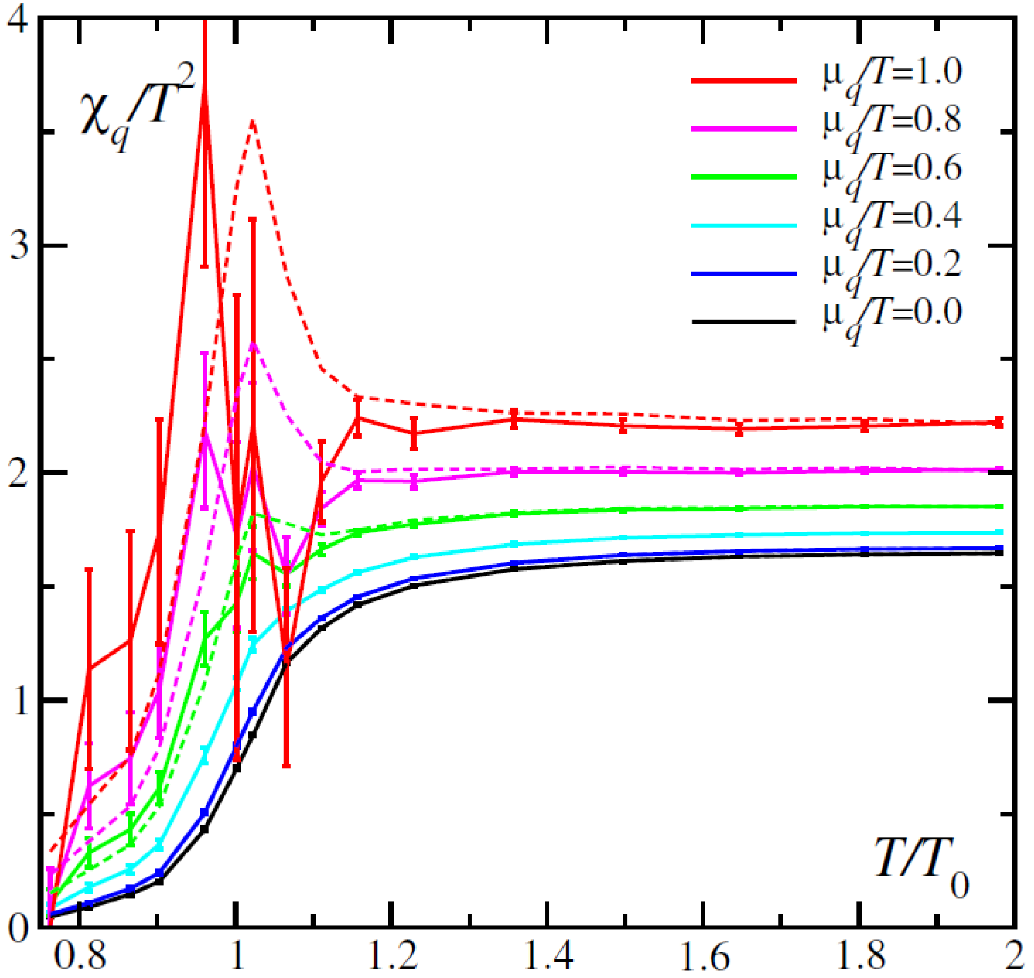
L. McLerran, Phys. Rev. D36, 3291 (1987).

Full/Quenched Lattice QCD





S. Gottlieb, W. Liu, D. Toussaint, R. L. Renken and R. L. Sugar, Phys. Rev. Lett. **59** (1987) 2247.

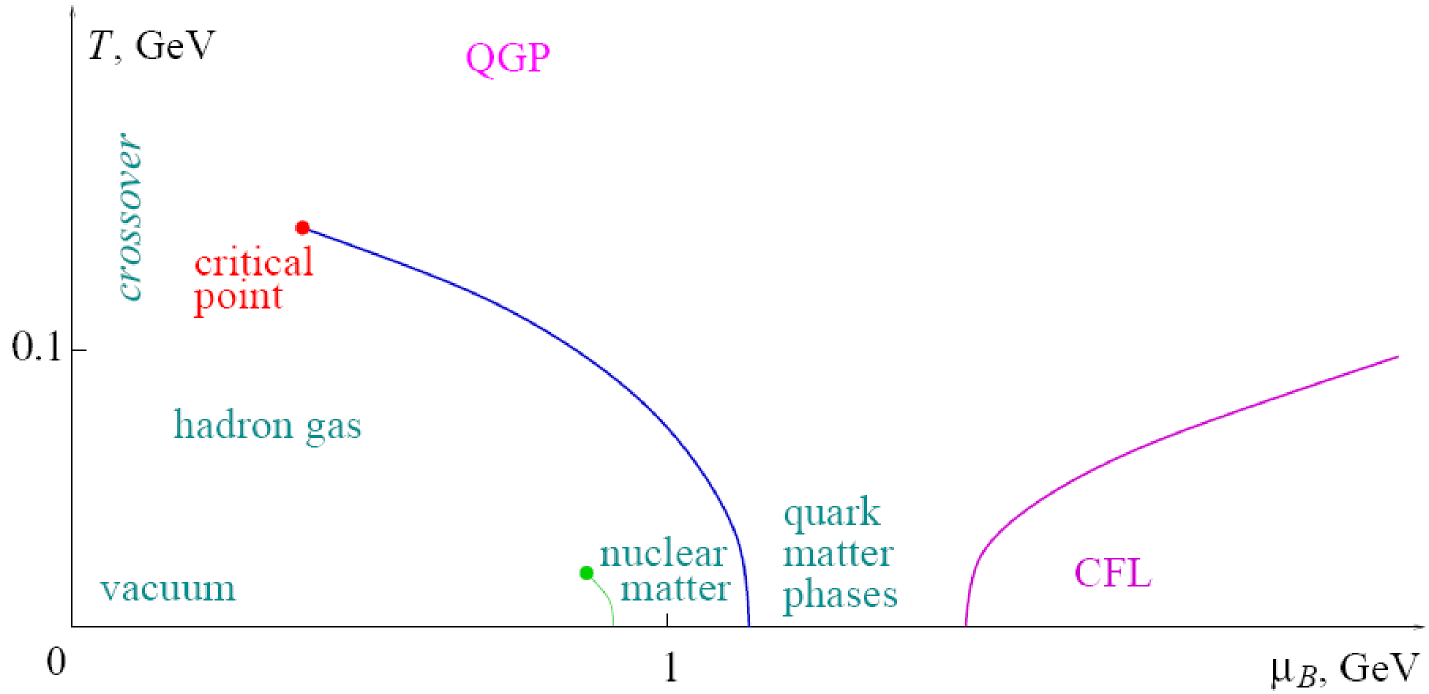


Signal of CEP?

(two flavor QCD)

Allton et al, Phys.Rev.D71:054508,2005

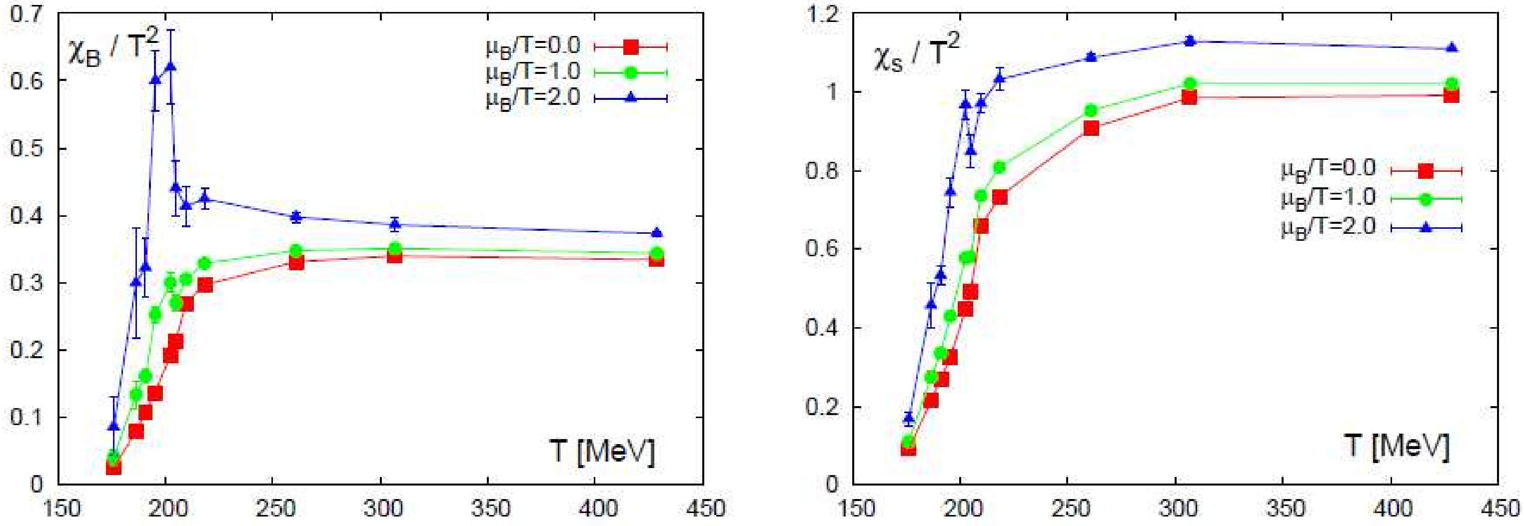
QCD phase diagram (building blocks)



Note: Crossover is firmly established by lattice (most recently Aoki *et al*)

M. Stephanov

Yukawa International Seminar (YKIS) 2006



Light and strange quark susceptibility from a 3-flavour lattice calculation at baryochemical potential $\mu_B = (3\mu_q) = 0$, T_c and 2 T_c.

Cheng et al, Phys.Rev.D79:074505,2009

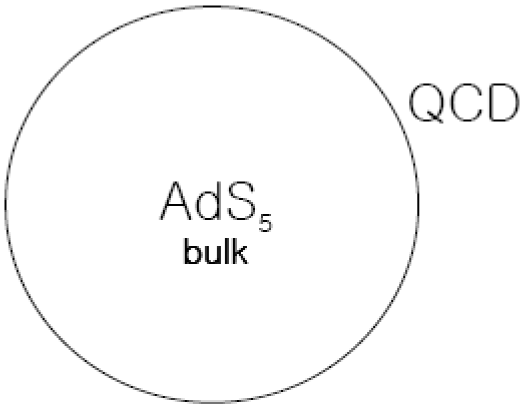
hQCD in a nutshell

String Theory in higher dim. ⇔ Gauge Theory in 4 dim.

Weakly coupled ⇔ Strongly coupled

Maldacena 98

AdS/QCD ?



$$\text{4D generating functional} : Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}\,,$$

$$\text{5D (classical) effective action} : \Gamma_5[\phi(x,z) = \phi_0(x)]\,; \phi_0(x) = \phi(x, z = 0).$$

$$\text{AdS/CFT correspondence} : Z_4 = \Gamma_5.$$

AdS/CFT Dictionary

- 4D CFT (QCD) ←→ 5D AdS
- 4D generating functional ←→ 5D (classical) effective action
- Operator ←→ 5D bulk field
- [Operator] ←→ 5D mass
- Current conservation ←→ gauge symmetry
- Large Q ←→ small z
- Confinement ←→ Compactified z
- Resonances ←→ Kaluza-Klein states

(a) Hard wall model

Let's start from 2-flavor QCD at low energy and

attempts to guess its 5D holographic dual,

AdS/CFT dictionaries.

☆ 5D field contents

Operator → 5D bulk field

$$\bar{q}_R q_L \longrightarrow \Phi(x,z)$$
$$\bar{q}_L \gamma^\mu q_L \longrightarrow L_M(x,z)$$
$$\bar{q}_R \gamma^\mu q_R \longrightarrow R_M(x,z)$$

[Operator] → 5D mass

$$(\Delta - p)(\Delta + p - 4) = m_5^2 \qquad m_\phi^2 = -3$$

☆ 5D Symmetry

Current conservation → gauge symmetry

$SU(2)_L X SU(2)_R$ gauge symmetry in AdS_5

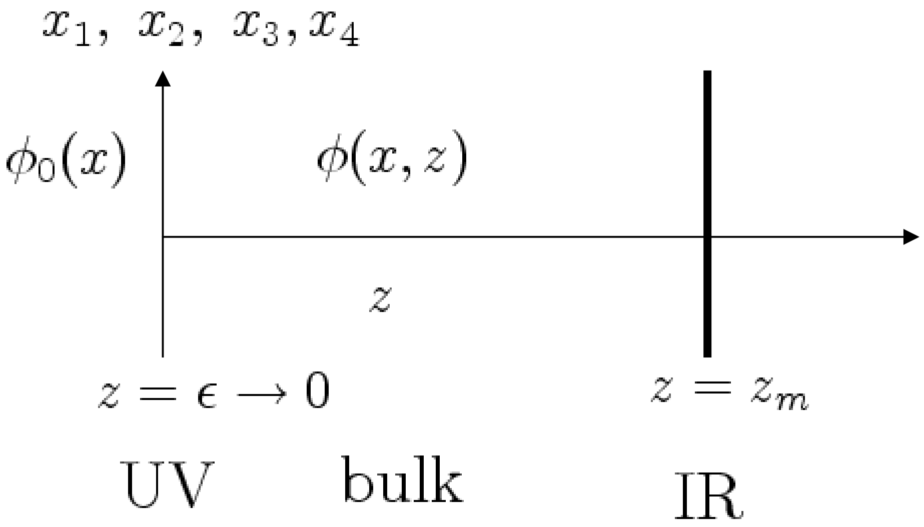
☆ Background: AdS_5

$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

☆ Confinement

Polchinski & Strassler, 2000

Confinement → IR cutoff in 5th direction



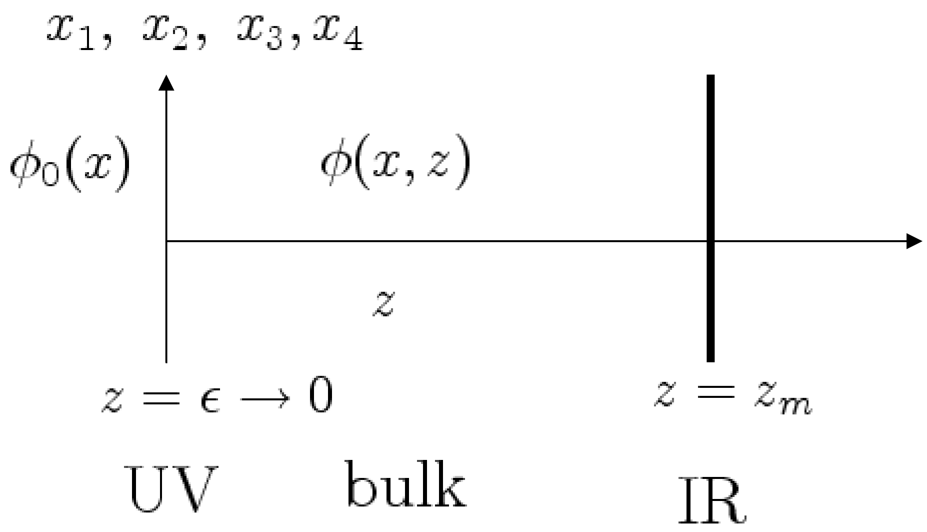
Hard wall model

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)

$$S_I = \int d^4x dz \sqrt{g} \mathcal{L}_5 \,,$$

$$\mathcal{L}_5 = \mathrm{Tr}\left[-\frac{1}{4g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) + |D_M\Phi|^2 - M_\Phi^2|\Phi|^2 \right] ,$$



$$V_M \sim L_M + R_M\,, \quad A_M \sim L_M - R_M\,,$$
$$\Phi = S e^{iP/v(z)}, \quad v(z) \equiv \langle S \rangle\,,$$
$$\Phi \leftrightarrow X, \quad v(z) \leftrightarrow X_0\,.$$

Example: vector-vector correlator

$$Z_4[\phi_0(x)] = \int D[\Phi] \exp\left[i S_4 + i \int \phi_0(x) O \right]$$

$$\langle O O \rangle \sim \frac{\delta Z_4[\phi_0]}{\delta \phi_0 \, \delta \phi_0}$$

4D: $\underset{\sim}{O} : \bar{q} \gamma^\mu t^a q \xleftrightarrow{\text{AdS/CFT}}$ 5D: $V^{\mu a}(x, z)$

$\downarrow$ Sol. of E.O.M, $z \to 0$

$V_0^{\mu a} \hookrightarrow \underset{\sim}{\phi_0}$

$$Z_4[\phi_0(x)] \xleftrightarrow{\text{AdS/CFT}} \underline{\Gamma_5[\phi_0(x)]}$$

$$\Downarrow$$

$$\underline{\underline{\Gamma_5[V_0]}} = Z_4[V_0]$$

[illegible]

$$S_5 = -\frac{1}{4g_5^2}\int d^5x \sqrt{g}\; V_{MN} V^{MN}$$

$M = (\mu, z)$

$$= -\frac{1}{4g_5^2}\int d^5x \frac{1}{z^5} z^4 \left(-2 V_{z\mu} V_{z\mu} + V_{\mu\nu} V^{\mu\nu}\right)$$

(z^4: $g^{zz} g^{\mu\mu}$)

gauge choice:

<u>$V_z = 0$</u>

$$= -\frac{1}{4g_5^2}\int d^5x \frac{1}{z}\left[-2(\partial_z V_\mu)^2 + (V_{\mu\nu})^2\right]$$

$\nearrow$ E.o.M

$$= -\frac{1}{2g_5^2}\int d^5x\, V_\mu \left[z\,\partial_z \left(\frac{1}{z}\partial_z\right)\eta_{\mu\nu} - \partial_4^2\,\eta_{\mu\nu} + \partial_\mu \partial_\nu \right] V_\nu \frac{1}{z}$$

$$+ \frac{1}{2g_5^2}\int d^4x \int dz\, \partial_z \left(\frac{1}{z} V_\mu \partial_z V_\mu\right)$$

$\rightarrow S_4$, $S_{boundary}$

E.o.M : $z\,\partial_z\left(\frac{1}{z}\partial_z V_\mu\right) - \left(\partial_4^2 V_\mu - \partial_\mu (\partial\cdot V)\right) = 0$

$\Downarrow$ F.T. in 4D.

$$z\,\partial_z\left(z^{-1}\partial_z V_\mu\right) + \left(q^2 \delta_\mu^\nu - q_\mu q^\nu\right) V_\nu = 0$$

$\Rightarrow V_\nu^\perp \cdot q^2$ transverse.

$$S_4 = -\frac{1}{2g_5^2}\int d^4x \left(\frac{1}{z} V_\mu \partial_z V^\mu\right)_{z=\epsilon}$$

$$V_\mu(q, z) = V(q, z) V_0^\mu(q) \quad , \quad V(q, \epsilon) = 1$$

↳ source term of the vector current $\sim \bar{q} \gamma_\mu t^a q$

$$S_4 = -\frac{1}{2 g_5^2} \int d^4x \, V_0^\mu(q) \frac{1}{z} \partial_z V(q, \epsilon) \, V_{0,\mu}(q)$$

<u>V-V correlator</u>

$$\int e^{iq\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(Q^2)$$

$Q^2 = -q^2$

i) S_b, S_4

$$\Pi_V(Q^2) = -\frac{1}{2 g_5^2} \ln Q^2$$

$$\therefore g_5^2 = \frac{12\pi^2}{N_c}$$

ii) OPE

$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2} \ln Q^2$$

Example: holographic deconfinement transition

1. thermal AdS:

$$ds^2 = L^2 \left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2} \right)$$

β' : the periodicity in the timedirection, (undetermined)

2. AdS black hole: $f(z) = 1 - \frac{z^4}{z_h^4}$ $\quad T = \frac{1}{\pi z_h}$

$$ds^2 = \frac{L^2}{z^2} \left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right) \qquad 0 \le t < \pi z_h$$

Transition between two backgrounds ⟷ (De)confinement transition

(b) Retarded Green's function
in hQCD

A way to obtain the retarded thermal Green's function

$$S_{\rm cl} = \frac{1}{2}\int du\, d^4x\, A(u)(\partial_u\phi)^2 + \cdots$$

↓

Solving the EoM of the bulk field with in–falling boundary condition

↓

$$\phi(u,q) = f_q(u)\phi_0(q)\,,$$

↓

$$G^R(q) = A(u)f_{-q}(u)\partial_u f_q(u)|_{u\to 0}\,.$$

An example

$$S = -\frac{1}{4g_{SG}^2}\int d^5x\sqrt{-g}\,F_{\mu\nu}^a F^{\mu\nu\;a}\,,$$

$$ds^2 = \frac{(\pi T R)^2}{u}\left(-f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{4u^2 f(u)}du^2 + R^2 d\Omega_5^2\,,$$

$$u = r_0^2/r^2 \qquad f(u) = 1 - u^2 \qquad T = r_0/\pi R^2$$

Infalling (incoming) wave: $e^{-i\omega t} f_p \sim e^{-i\omega(t+r_\star)}$, outgoing wave : $e^{-i\omega t} f_p^\star \sim e^{-i\omega(t-r_\star)}$, where $r_\star = log(1-u)/(4\pi T)$, $u \to 1$ means $r_\star \to -\infty$.

$$A_i = \int \frac{d^4q}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{q}\cdot\boldsymbol{x}} A_i(q,u) \,.$$

$$q = (\omega, 0, 0, q)$$

$$\mathfrak{w} = \frac{\omega}{2\pi T} \,, \qquad \mathfrak{q} = \frac{q}{2\pi T} \,,$$

the five-dimensional Maxwell equations,

$$\frac{1}{\sqrt{-g}}\partial_\nu[\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)] = 0\,,$$

reduce to the following set of the ordinary differential equations

$$\mathfrak{w} A_t' + \mathfrak{q} f\, A_z' = 0\,,$$

$$A_t'' - \frac{1}{uf}\left(\mathfrak{q}^2 A_t + \mathfrak{w}\mathfrak{q} A_z\right) = 0\,,$$

dependent ⟶ $$A_z'' + \frac{f'}{f}A_z' + \frac{1}{uf^2}\left(\mathfrak{w}^2 A_z + \mathfrak{w}\mathfrak{q} A_t\right) = 0\,,$$

$$A_\alpha'' + \frac{f'}{f}A_\alpha' + \frac{1}{uf}\left(\frac{\mathfrak{w}^2}{f} - \mathfrak{q}^2\right) A_\alpha = 0.$$

From the first and second equations,

$$A_t''' + \frac{(uf)'}{uf} A_t'' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f(u)}{uf^2} A_t' = 0$$

In-falling BC $\longrightarrow$ $A_t' = (1-u)^\nu F(u), \quad \nu = -i\frac{\omega}{4\pi T},$

Now we are to solve the EoM for F(u) in the limit of long-wave length and low-frequency.

$$F'' + \left(\frac{1-3u^2}{uf} + \frac{i\mathfrak{w}}{1-u}\right) F' + \frac{i\mathfrak{w}(1+2u)}{2uf} F + \frac{\mathfrak{w}^2[4-u(1+u)^2]}{4uf^2} F - \frac{\mathfrak{q}^2}{uf} F = 0\,.$$

$$F(u) = F_0 + \mathfrak{w} F_1 + \mathfrak{q}^2 G_1 + \mathfrak{w}^2 F_2 + \mathfrak{w}\mathfrak{q}^2 H_1 + \mathfrak{q}^4 G_2 + \cdots\,.$$

$$F_0 = C\,, \qquad F_1 = \frac{iC}{2}\ln\frac{2u^2}{1+u}\,, \qquad G_1 = C\ln\frac{1+u}{2u}\,.$$

Putting A_t' back into the EoM and taking $u \to 0$ limit with the BCs

$$\lim_{u\to 0} A_t(u) = A_t^0\,, \qquad \lim_{u\to 0} A_z(u) = A_z^0\,.$$

This determines the constant C in terms of A_t^0, A_z^0:

$$C = \frac{\mathfrak{q}^2 A_t^0 + \mathfrak{w}\mathfrak{q} A_z^0}{Q(\mathfrak{w},\mathfrak{q})}\,,$$

where $Q(\mathfrak{w},\mathfrak{q})$ has the following expansion over the small arguments,

$$Q(\mathfrak{w},\mathfrak{q}) = i\mathfrak{w} - \mathfrak{q}^2 + O(\mathfrak{w}^2, \mathfrak{w}\mathfrak{q}^2, \mathfrak{q}^4)\,.$$

Then, we obtain

$$A_t' = (\mathfrak{q}^2 A_t^0 + \mathfrak{w}\mathfrak{q} A_z^0)\ln\epsilon + \frac{\mathfrak{q}^2 A_t^0 + \mathfrak{w}\mathfrak{q} A_z^0}{i\mathfrak{w} - \mathfrak{q}^2}$$

On the other hand, the terms in the action which contain two derivatives with respect to u are

$$\begin{aligned} S &= -\frac{N^2}{32\pi^2 R}\int du\, d^4x\, \sqrt{-g}\, g^{uu} g^{ij}\partial_u A_i \partial_u A_j + \cdots \\ &= \frac{N^2 T^2}{16}\int du\, d^4x\, [A_t'^2 - f(A_x'^2 + A_y'^2 + A_z'^2)] + \cdots \end{aligned}$$

Following the procedure, we obtain

$$G_{tt}^{ab} = \frac{N^2 T q^2 \, \delta^{ab}}{16\pi(i\omega - Dq^2)} + \cdots ,$$

where $\cdots$ denotes corrections of order $\mathfrak{w}^2$, $\mathfrak{w}\mathfrak{q}^2$ or $\mathfrak{q}^4$, and

diffusion constant $\longrightarrow$ $D = \dfrac{1}{2\pi T}$.

G. Policastro, D. T. Son, and O. Starinets, JHEP09 (2002) 043

qSUS with finite chemical potential

$$\chi_q(T, \mu_q) = -\lim_{k \to 0} \mathrm{Re}\, \Pi^{\mathrm{R}}_{00}(0, k) .$$

T. Kunihiro, Phys. Lett. B 271, (1991) 395

Reissner-Nordström-AdS background

An AdS BH with U(1) charge

$$S=\frac{1}{2\kappa^2}\int \mathrm{d}^5x\sqrt{-g}\Big(R-2\Lambda\Big)-\frac{1}{4e^2}\int \mathrm{d}^5x\sqrt{-g}\mathcal{F}_{mn}\mathcal{F}^{mn},$$

$$\mathrm{d}s^2=\frac{r^2}{l^2}\Big(-f(r)(\mathrm{d}t)^2+(\mathrm{d}\vec{x})^2\Big)+\frac{l^2}{r^2f(r)}(\mathrm{d}r)^2,$$

$$\mathcal{A}_t=-\frac{Q}{r^2}+\mu,$$

$$f(r)=1-\frac{ml^2}{r^4}+\frac{q^2l^2}{r^6},\qquad \Lambda=-\frac{6}{l^2},\qquad e^2=\frac{2Q^2}{3q^2}\kappa^2,\quad \mu=\frac{Q}{r_+^2}.$$

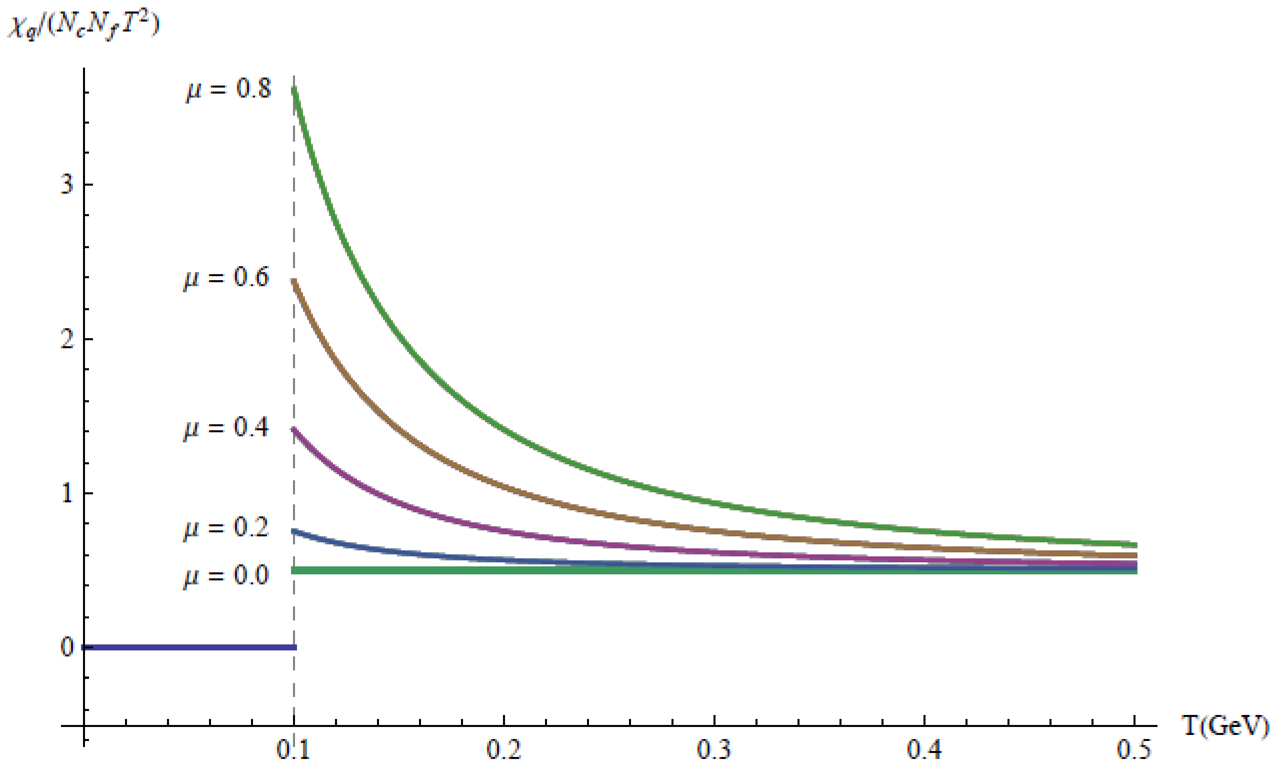


Figure 1: $\chi_q/(N_c N_f T^2)$ in the hard wall model for varying μ(GeV) with $N_c = 3$ and $N_f = 2$.

Y. Kim, Y. Matsuo, W. Sim, S. Takeuchi, and T. Tsukioka, hep-th/10015343
(hard wall, soft wall, D3/D7, D4/D8 with chemical potential or magnetic field)

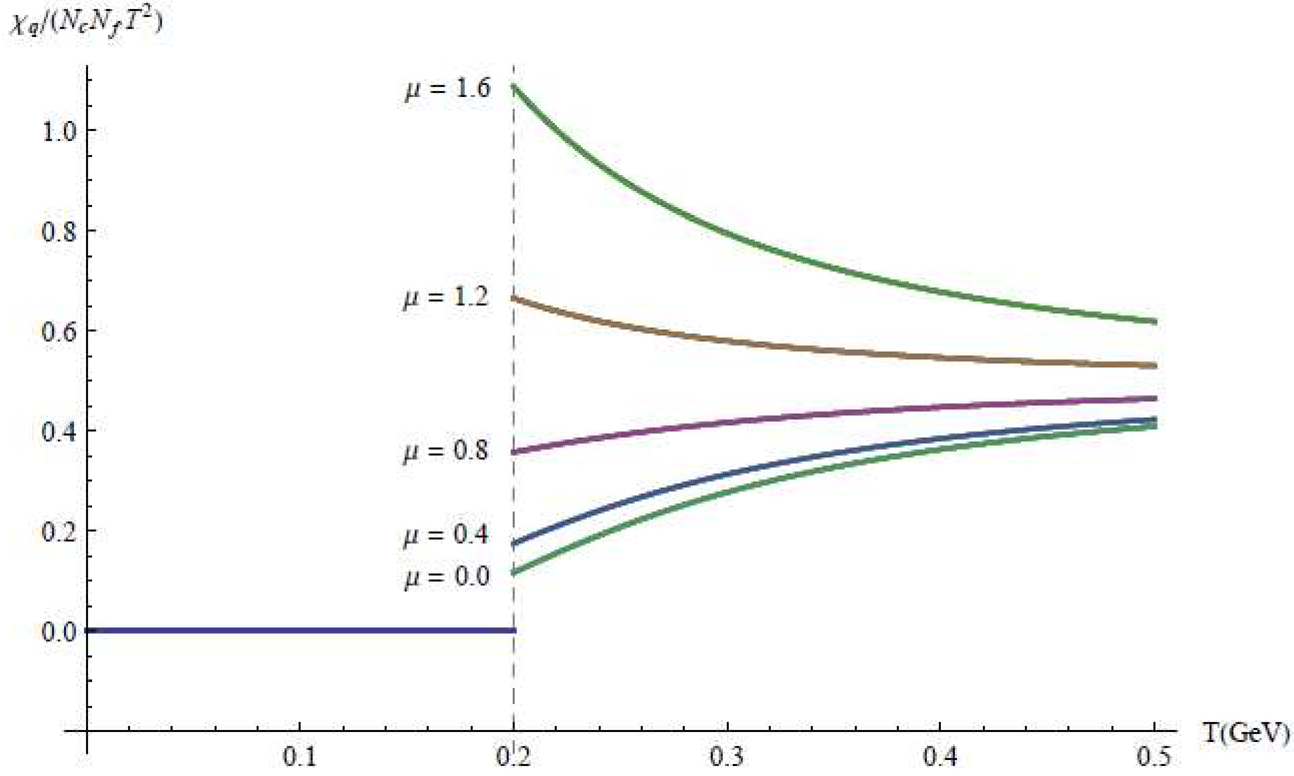


Figure 2: $\chi_q/(N_cN_fT^2)$ in the soft wall model for varying μ(GeV) with $N_c = 3$ and $N_f = 2$.

T/T_c	c_2		$c_4 \times 10$		$c_6 \times 10^2$		$c_8 \times 10^3$
	A	lattice	A	lattice	A	lattice	A
1.00	0.350*	0.350	0.0702	2.13	0.00256	−5.00	−0.000814
1.02	0.373	0.423	0.0726	2.26	0.00223	−4.49	−0.000761
1.07	0.431	0.582	0.0782	1.42	0.00144	−5.73	−0.000611
1.11	0.476	0.658	0.0821	0.951	0.000839	−1.65	−0.000483
1.16	0.531	0.709	0.0863	0.763	0.000150	−0.31	−0.000322
1.23	0.603	0.752	0.0911	0.667	−0.000697	−0.44	−0.000107
1.36	0.723	0.788	0.0976	0.572	−0.00192	−0.09	0.000232
1.50	0.831	0.806	0.102	0.539	−0.00284	−0.17	0.000504
1.65	0.927	0.816	0.105	0.499	−0.00349	−0.13	0.000708
1.81	1.01	0.820	0.108	0.497	−0.00396	−0.11	0.000855
1.98	1.08	0.823	0.109	0.473	−0.00428	−0.03	0.000962

Table 1: Results with 5D gauge coupling from D3/D7, case A, compared with lattice data

$$\chi_q/T^2 = \sum_n 2n(2n-1)c_{2n}(\mu/T)^{2(n-1)}.$$

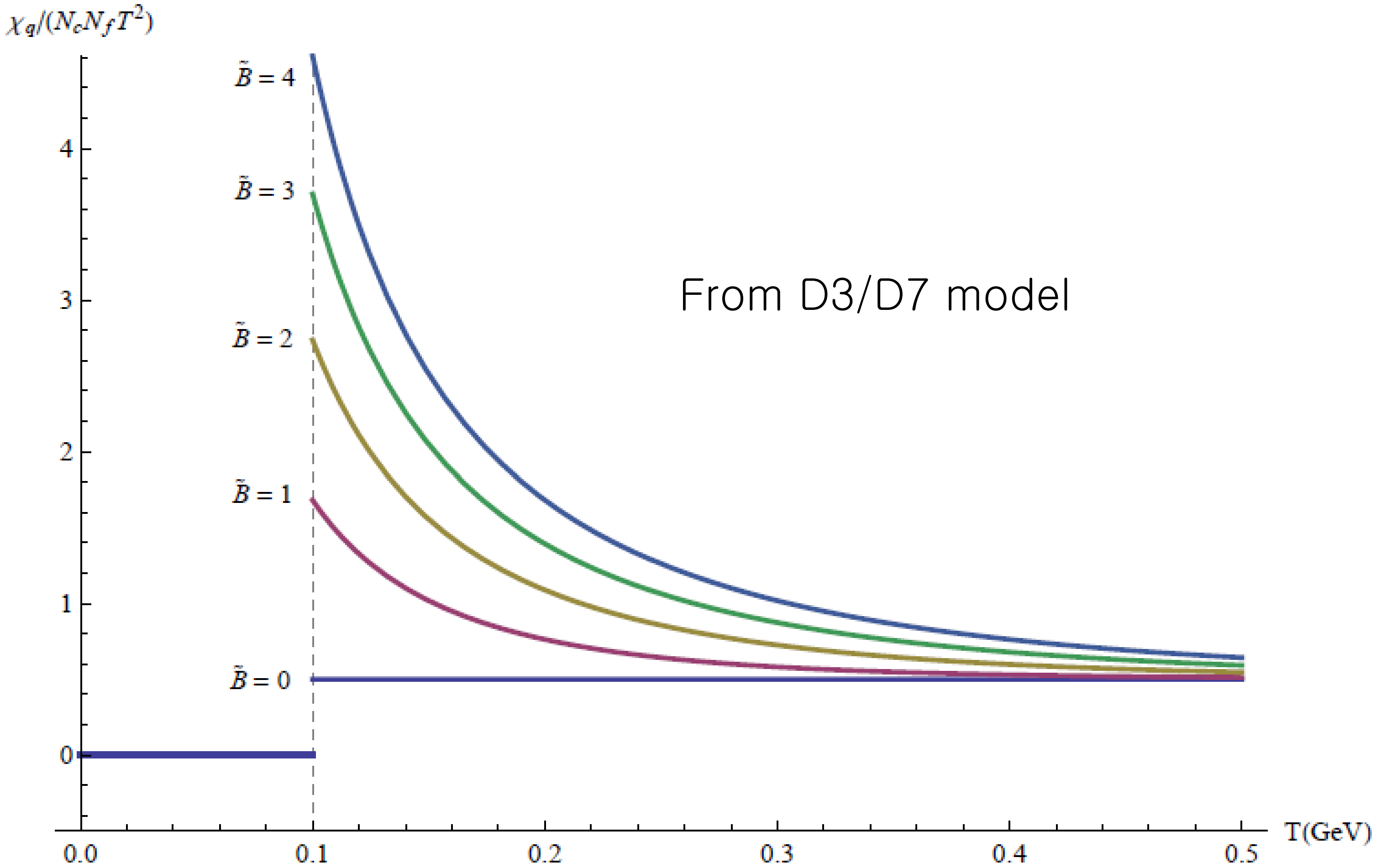


Figure 3: $\chi_q/(N_c N_f T^2)$ for varying $\widetilde{B} = 2\pi\alpha' B$ with $Nc = 3$ and $N_f = 2$.

First, for the case of D4/D8,

$$
\begin{aligned}
\chi_{D4/D8} &\sim l_s^{-9}\alpha'^2 R^4 \left(\int_{U_T}^{\infty} \frac{1}{(U/R)\sqrt{(U/R)^3 + (2\pi\alpha' B)^2}} \right)^{-1} \\
B = 0 \rightarrow \quad &\sim l_s^{-9}\alpha'^2 R^4 (R^{-5/2} U_T^{3/2}) \\
U_T \sim R^3 T^2 \rightarrow \quad &\sim l_s^{-9}\alpha'^2 R^4 (R^2 T^3).
\end{aligned}
$$

Next, for the case of D3/D7,

$$
\begin{aligned}
\chi_{D3/D7} &\sim l_s^{-8}\alpha'^2 R^3 \left(\int_{U_T}^{\infty} \frac{1}{(U/R)\sqrt{(U/R)^4 + (2\pi\alpha' B)^2}} \right)^{-1} \\
B = 0 \rightarrow \quad &\sim l_s^{-8}\alpha'^2 R^3 (R^{-3} U_T^{2}) \\
U_T \sim R^2 T^1 \rightarrow \quad &\sim l_s^{-8}\alpha'^2 R^3 (R^1 T^2).
\end{aligned}
$$

Discussion

- Finite quark mass effect? (in progress)
- RN-AdS is describing QGP with non-zero baryon number density?
- What is the gravity background dual to QGP with/without density???
- Generic problem is of course to collect all large Nc leading corrections in a consisent way.