Four-nucleon interactions from holography

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based on Youngman Kim, DY, Piljin Yi (1111.3118)

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Introduction: The effective Lagrangian

- Four-nucleon contact interactions in the effective chiral Lagrangian Weinberg (1990) up to Q^2 order:

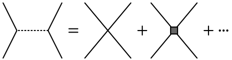
  Ordonez et al. (1996)

$$\mathcal{L} = -\frac{1}{2} C_S O_S - \frac{1}{2} C_T O_T - \sum_{i=1}^{14} C_i' O_i \, .$$

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$
O_1	$(N^\dagger \nabla N)^2 + \mathrm{h.c.}$
O_2	$(N^\dagger \nabla N) \cdot (\nabla N^\dagger N)$
O_3	$(N^\dagger N)(N^\dagger \nabla^2 N) + \mathrm{h.c.}$
O_4	$i\,(N^\dagger \nabla N) \cdot (\nabla N^\dagger \times \boldsymbol{\sigma} N) + \mathrm{h.c.}$
O_5	$i\,(N^\dagger N)(\nabla N^\dagger \cdot \boldsymbol{\sigma} \times \nabla N)$
O_6	$i\,(N^\dagger \boldsymbol{\sigma} N) \cdot (\nabla N^\dagger \times \nabla N)$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \nabla N)(N^\dagger \boldsymbol{\sigma} \cdot \nabla N) + \mathrm{h.c.}$
O_8	$(N^\dagger \sigma^i \partial_j N)(N^\dagger \sigma^j \partial_i N) + \mathrm{h.c.}$
O_9	$(N^\dagger \sigma^i \partial_j N)(N^\dagger \sigma^i \partial_j N) + \mathrm{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \nabla N)(\nabla N^\dagger \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^i \partial_j N)(\partial_i N^\dagger \sigma^j N)$
O_{12}	$(N^\dagger \sigma^i \partial_j N)(\partial_j N^\dagger \sigma^i N)$
O_{13}	$(N^\dagger \sigma^i N)(\partial_j N^\dagger \sigma^j \partial_i N) + \mathrm{h.c.}$
O_{14}	$2(N^\dagger \sigma^i N)(\partial_j N^\dagger \sigma^i \partial_j N)$

- The low energy constants (LECs)
  - leading (Q^0) order: C_S, C_T
  - next (Q^2) order: C_is

Meson exchange interactions



- Outline of the process

  pre. AdS/CFT, D4/D8/$\overline{\text{D8}}$, holographic baryon, ...
  1. 5d meson and baryon with cubic interactions
  2. down into 4d and carrying out cubic couplings g
  3. integrating out mesons
  4. non-relativistic reduction (+constraints)
  5. matching $4N$ operators with LECs

- 5d interaction Hong-Rho-Yee-Yi (2007):

$$-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} \quad \text{and} \quad \bar{\mathcal{B}}\gamma^{mn} F_{mn}\mathcal{B}$$

  will give the 4d cubic couplings

$$g_V \bar{\mathcal{N}}\gamma^\mu \omega_\mu \mathcal{N}\,,\; g_V \bar{\mathcal{N}}\gamma^{\mu\nu}\partial_\mu \rho_\nu \mathcal{N}\,, \cdots\,.$$

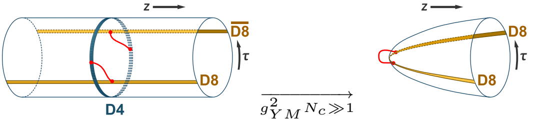
- For example,

$$g_V^{(k)singlet} = \int dw\; |f_+(w)|^2\, \psi_{(2k-1)}(w)$$

  - $\psi_{(n)}(w)$: profile functions of meson fields,
  - $f_+(w)$: profile functions of baryon fields.

5d Lagrangian from D4/D8/$\overline{\text{D8}}$

- AdS/CFT correspondence Maldacena (1997)
  is the duality between
  - $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $SU(N_c)$
  - and closed string theory in $AdS_5 \times S^5$.
- D4/D8/$\overline{\text{D8}}$ model Sakai-Sugimoto (2004)
  embodies this duality as
  - the five dim gauge theory is fixed by the brane configuration,
  - which represents $U(N_c)$ with N_f massless flavors.



This construction provides holographic manifestation of the chiral symmetry breaking.

- Meson sector: Sakai-Sugimoto (2004)
  5d $U(N_f)$ gauge theory on N_f D8

$$-\frac{1}{4}\int d^4x dw \;\; \frac{1}{e(w)^2}\,\mathrm{tr}\mathcal{F}^2 + \frac{N_c}{24\pi^2}\int_{4+1}\omega_5(\mathcal{A})$$

$$\mathcal{A}_\mu(x,w) = i\,[U^{-1/2},\partial_\mu U^{1/2}]/2 + i\{U^{-1/2},\partial_\mu U^{1/2}\}\psi_0(w) + \sum_n v_\mu^{(n)}(x)\psi_{(n)}(w)\,.$$

$e^{2i\pi(x)/f_\pi} = U(x) = e^{i\int \mathcal{A}_5(x,w)dw}$

- Baryon sector: Hong-Rho-Yee-Yi (2007)
  flavor soliton, baryon as wrapped D4 brane.
  $\rightarrow$ effective theory of isospin 1/2 baryons in 5d

$$+\int d^4x\,dw\left[-i\bar{\mathcal{B}}\gamma^m D_m\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B}\right]$$

where $D_m \equiv \partial_m - i(N_c\mathcal{A}_m^{U(1)} + A_m)$.

$$\mathcal{B}(x,w) = \mathcal{N}_+(x)f_+(w) + \mathcal{N}_-(x)f_-(w)\,,$$

4d Lagrangian

By integrating out the holographic direction w, we get Kim-Lee-Yi (2009)

$$\int d^4x\, \mathcal{L}_4 = \int d^4x \left(-i\bar{\mathcal{N}}\gamma^\mu\partial_\mu\mathcal{N} - im_{\mathcal{N}}\bar{\mathcal{N}}\mathcal{N} + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}}\right) ,$$

$$\begin{aligned}
\mathcal{L}_{\text{vector}} = &- \sum_{k\geq 1} \frac{g_V^{(k)triplet}}{2}\bar{\mathcal{N}}\gamma^\mu\rho_\mu^{(k)}\mathcal{N} - \sum_{k\geq 1}\frac{N_c g_V^{(k)singlet}}{2}\bar{\mathcal{N}}\gamma^\mu\omega_\mu^{(k)}\mathcal{N} \\
&+ \sum_{k\geq 1}\frac{g_{dV}^{(k)triplet}}{2}\bar{\mathcal{N}}\gamma^{\mu\nu}\partial_\mu\rho_\nu^{(k)}\mathcal{N} + \cdots ,\\
\mathcal{L}_{\text{axial}} = &\frac{g_A^{triplet}}{2f_\pi}\bar{\mathcal{N}}\gamma^\mu\gamma^5\partial_\mu\pi\mathcal{N} + \frac{N_c g_A^{singlet}}{2f_\pi}\bar{\mathcal{N}}\gamma^\mu\gamma^5\partial_\mu\eta'\mathcal{N} \\
&- \sum_{k\geq 1}\frac{g_A^{(k)triplet}}{2}\bar{\mathcal{N}}\gamma^\mu\gamma^5 a_\mu^{(k)}\mathcal{N} - \sum_{k\geq 1}\frac{N_c g_A^{(k)singlet}}{2}\bar{\mathcal{N}}\gamma^\mu\gamma^5 f_\mu^{(k)}\mathcal{N} + \cdots
\end{aligned}$$

with $\pi = \pi^a\tau^a$.

The couplings are

$$g_V^{(k)triplet} = \int_{-w_{max}}^{w_{max}} dw\, |f_+(w)|^2\, \psi_{(2k-1)}(w)$$
$$+ 2\int_{-w_{max}}^{w_{max}} dw \left(g_5(w) \frac{\rho_{baryon}^2}{e(w)^2} \right) |f_+(w)|^2\, \partial_w \psi_{(2k-1)}(w)\,,$$

$$g_A^{(k)triplet} = 2\int_{-w_{max}}^{w_{max}} dw \left(g_5(w) \frac{\rho_{baryon}^2}{e(w)^2} \right) |f_+(w)|^2\, \partial_w \psi_{(2k)}(w)$$
$$+ \int_{-w_{max}}^{w_{max}} dw\, |f_+(w)|^2\, \psi_{(2k)}(w)\,,$$

$$g_{dV}^{(k)triplet} = 2\int_{-w_{max}}^{w_{max}} dw \left(g_5(w) \frac{\rho_{baryon}^2}{e(w)^2} \right) f_-^*(w) f_+(w) \psi_{(2k-1)}(w)\,,$$

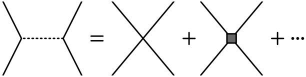
$$g_V^{(k)singlet} = \int_{-w_{max}}^{w_{max}} dw\, |f_+(w)|^2\, \psi_{(2k-1)}(w)\,,$$

$$g_A^{(k)singlet} = \int_{-w_{max}}^{w_{max}} dw\, |f_+(w)|^2\, \psi_{(2k)}(w)\,,$$

$$g_A^{singlet} = 2\int_{-w_{max}}^{w_{max}} dw\, |f_+(w)|^2\, \psi_{(0)}(w)\,.$$

Integrating out mesons

- eom of meson v $\rightarrow$ cubic term $v\mathcal{N}\mathcal{N}$ becomes $\mathcal{N}\mathcal{N}\mathcal{N}\mathcal{N}$



- For example, isosinglet vector meson ω case, the four-point interaction up to Q^2 order:

$$\mathcal{L}_\omega = \sum_{k\geq 1} \frac{1}{2m^2_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \bar{\mathcal{N}}\gamma^\mu\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\mathcal{N} + \sum_{k\geq 1} \frac{1}{2m^4_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \bar{\mathcal{N}}\gamma^\mu\mathcal{N}\partial^2\left(\bar{\mathcal{N}}\gamma_\mu\mathcal{N}\right)$$

- The relativistic 4$\mathcal{N}$ operators after integrating out mesons:

$$\bar{\mathcal{N}}\gamma^\mu\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\mathcal{N}\,, \qquad \bar{\mathcal{N}}\gamma^\mu\mathcal{N}\partial^2\left(\bar{\mathcal{N}}\gamma_\mu\mathcal{N}\right)\,,$$
$$\bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}\,, \qquad \bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N}\partial^2\left(\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}\right)\,,$$
$$\partial_\mu\left(\bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N}\right)\partial_\nu\left(\bar{\mathcal{N}}\gamma^\nu\gamma^5\mathcal{N}\right)\,, \qquad \bar{\mathcal{N}}\gamma_\mu\mathcal{N}\partial_\nu\left(\bar{\mathcal{N}}\gamma^{\nu\mu}\mathcal{N}\right)\,,$$
$$\partial_\nu\left(\bar{\mathcal{N}}\gamma^\nu{}_\mu\mathcal{N}\right)\partial_\lambda\left(\bar{\mathcal{N}}\gamma^{\lambda\mu}\mathcal{N}\right)$$

Non-relativistic limit

- We expand the relativistic fermion field $\mathcal{N}$

$$\mathcal{N}(x) = \begin{pmatrix} N(x) + \frac{1}{8m_{\mathcal{N}}^2}\nabla^2 N(x) \\ \frac{1}{2m_{\mathcal{N}}}\boldsymbol{\sigma}\cdot\nabla N(x) \end{pmatrix} + \mathcal{O}(Q^3)$$

where N is the two-component spinor.

- For example,

$$\begin{aligned}
\bar{\mathcal{N}}\gamma^\mu\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\mathcal{N} \rightarrow & -N^\dagger N N^\dagger N + \frac{1}{4m_{\mathcal{N}}^2}\Big(4(N^\dagger\nabla N)\cdot(\nabla N^\dagger N) \\
& + 2i\,(N^\dagger N)(\nabla N^\dagger\cdot\boldsymbol{\sigma}\times\nabla N) - 4i\,(N^\dagger\boldsymbol{\sigma}N)\cdot(\nabla N^\dagger\times\nabla N) \\
& - (N^\dagger\boldsymbol{\sigma}\cdot\nabla N)(N^\dagger\boldsymbol{\sigma}\cdot\nabla N) + \text{h.c.} \\
& + (N^\dagger\sigma^i\partial_j N)(N^\dagger\sigma^i\partial_j N) + \text{h.c.} \\
& - 2(N^\dagger\boldsymbol{\sigma}\cdot\nabla N)(\nabla N^\dagger\cdot\boldsymbol{\sigma}N) + 2(N^\dagger\sigma^i\partial_j N)(\partial_j N^\dagger\sigma^i N)\Big) \\
= & -O_S + \frac{1}{4m_{\mathcal{N}}^2}\left(4O_2 + 2O_5 - 4O_6 - O_7 + O_9 - 2O_{10} + 2O_{12}\right)
\end{aligned}$$

Relativistic constraints

- From the beginning, the four-nucleon Lagrangian up to Q^2 order was

$$\mathcal{L} = -\frac{1}{2}C_S O_S - \frac{1}{2}C_T O_T - \sum_{i=1}^{14} C'_i O_i \,.$$

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$
O_1	$(N^\dagger \nabla N)^2 + \text{h.c.}$
O_2	$(N^\dagger \nabla N)\cdot(\nabla N^\dagger N)$
O_3	$(N^\dagger N)(N^\dagger \nabla^2 N) + \text{h.c.}$
O_4	$i\,(N^\dagger \nabla N)\cdot(\nabla N^\dagger \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i\,(N^\dagger N)(\nabla N^\dagger \cdot \boldsymbol{\sigma} \times \nabla N)$
O_6	$i\,(N^\dagger \boldsymbol{\sigma} N)\cdot(\nabla N^\dagger \times \nabla N)$
O_7	$(N^\dagger \boldsymbol{\sigma}\cdot\nabla N)(N^\dagger \boldsymbol{\sigma}\cdot\nabla N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^i \partial_j N)(N^\dagger \sigma^j \partial_i N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^i \partial_j N)(N^\dagger \sigma^i \partial_j N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma}\cdot\nabla N)(\nabla N^\dagger \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^i \partial_j N)(\partial_i N^\dagger \sigma^j N)$
O_{12}	$(N^\dagger \sigma^i \partial_j N)(\partial_j N^\dagger \sigma^i N)$
O_{13}	$(N^\dagger \sigma^i N)(\partial_j N^\dagger \sigma^j \partial_i N) + \text{h.c.}$
O_{14}	$2(N^\dagger \sigma^i N)(\partial_j N^\dagger \sigma^i \partial_j N)$

- The underlying Lorentz symmetry constrains that only nine (2+7) linearly independent combinations appear up to Q^2. Girlanda et al. (2010)

$$\mathcal{A}_S = O_S + \frac{1}{4m^2}(O_1 + O_3 + O_5 + O_6)\,,$$

$$\mathcal{A}_T = O_T - \frac{1}{4m^2}(O_5 + O_6 - O_7 + O_8 + 2\,O_{12} + O_{14})\,,$$

$$\mathcal{A}_1 = O_1 + 2\,O_2\,, \mathcal{A}_2 = 2\,O_2 + O_3\,, \mathcal{A}_3 = O_9 + 2\,O_{12}\,,$$

$$\mathcal{A}_4 = O_9 + O_{14}\,, \mathcal{A}_5 = O_5 - O_6\,,$$

$$\mathcal{A}_6 = O_7 + 2\,O_{10}\,, \mathcal{A}_7 = O_7 + O_8 + 2\,O_{13}$$

- Then the effective Lagrangian

$$\mathcal{L} = -\frac{1}{2}C_S O_S - \frac{1}{2}C_T O_T - \sum_{i=1}^{14} C'_i O_i\,.$$

can be written as

$$\begin{aligned}\mathcal{L} =& -\frac{1}{2}C_S\mathcal{A}_S - \frac{1}{2}C_T\mathcal{A}_T \\ & -\frac{1}{2}C_1\mathcal{A}_1 + \frac{1}{8}C_2\mathcal{A}_2 - \frac{1}{2}C_3\mathcal{A}_3 - \frac{1}{8}C_4\mathcal{A}_4 \\ & -\frac{1}{4}C_5\mathcal{A}_5 - \frac{1}{2}C_6\mathcal{A}_6 - \frac{1}{16}C_7\mathcal{A}_7\,.\end{aligned}$$

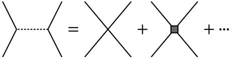
(definition of C_i's)

- Also we get

$$\begin{aligned}\bar{\mathcal{N}}\gamma^\mu\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\mathcal{N} \rightarrow & -O_S + \frac{1}{4m_\mathcal{N}^2}\left(4O_2 + 2O_5 - 4O_6 - O_7 + O_9 - 2O_{10} + 2O_{12}\right) \\ &= -\mathcal{A}_S + \frac{1}{4m_\mathcal{N}^2}\left(\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + 3\mathcal{A}_5 - \mathcal{A}_6\right)\,.\end{aligned}$$

- The whole structures we encounter are

$$
\begin{aligned}
\bar{\mathcal{N}}\gamma^{\mu}\mathcal{N}\bar{\mathcal{N}}\gamma_{\mu}\mathcal{N} &\to -O_S + \frac{1}{4m_{\mathcal{N}}^2}\left(4O_2 + 2O_5 - 4O_6 - O_7 + O_9 - 2O_{10} + 2O_{12}\right) \\
&= -\mathcal{A}_S + \frac{1}{4m_{\mathcal{N}}^2}\left(\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + 3\mathcal{A}_5 - \mathcal{A}_6\right) , \\
\bar{\mathcal{N}}\gamma^{\mu}\mathcal{N}\partial^2\left(\bar{\mathcal{N}}\gamma_{\mu}\mathcal{N}\right) &\to O_1 + 2O_2 = \mathcal{A}_1 , \\
\bar{\mathcal{N}}\gamma^{\mu}\gamma^5\mathcal{N}\bar{\mathcal{N}}\gamma_{\mu}\gamma^5\mathcal{N} &\to O_T + \frac{1}{4m_{\mathcal{N}}^2}\left(-2O_6 + O_7 - O_9 - 2O_{10} - 2O_{12} + 2O_{13} - 2O_{14}\right) \\
&= \mathcal{A}_T + \frac{1}{4m_{\mathcal{N}}^2}\left(-\mathcal{A}_4 + \mathcal{A}_5 - \mathcal{A}_6 + \mathcal{A}_7\right) , \\
\bar{\mathcal{N}}\gamma^{\mu}\gamma^5\mathcal{N}\partial^2\left(\bar{\mathcal{N}}\gamma_{\mu}\gamma^5\mathcal{N}\right) &\to O_9 + 2O_{12} = \mathcal{A}_3 , \\
\partial_{\mu}\left(\bar{\mathcal{N}}\gamma^{\mu}\gamma^5\mathcal{N}\right)\partial_{\nu}\left(\bar{\mathcal{N}}\gamma^{\nu}\gamma^5\mathcal{N}\right) &\to O_7 + 2O_{10} = \mathcal{A}_6 , \\
\bar{\mathcal{N}}\gamma_{\mu}\mathcal{N}\partial_{\nu}\left(\bar{\mathcal{N}}\gamma^{\nu\mu}\mathcal{N}\right) &\to \frac{1}{2m_{\mathcal{N}}}\left(O_1 + 2O_2 + 2O_5 - 2O_6 - O_7 + O_9 - 2O_{10} + 2O_{12}\right) \\
&= \frac{1}{2m_{\mathcal{N}}}\left(\mathcal{A}_1 + \mathcal{A}_3 - 2\mathcal{A}_5 - \mathcal{A}_6\right) , \\
\partial_{\nu}\left(\bar{\mathcal{N}}\gamma^{\nu}{}_{\mu}\mathcal{N}\right)\partial_{\lambda}\left(\bar{\mathcal{N}}\gamma^{\lambda\mu}\mathcal{N}\right) &\to -O_7 + O_9 - 2O_{10} + 2O_{12} = \mathcal{A}_3 - \mathcal{A}_6 .
\end{aligned}
$$

- Finally we get  for ω,

$$\mathcal{L}_\omega = \sum_{k\geq1} \frac{1}{2m^2_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \bar{\mathcal{N}}\gamma^\mu \mathcal{N} \bar{\mathcal{N}} \gamma_\mu \mathcal{N}$$

$$+ \sum_{k\geq1} \frac{1}{2m^4_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \bar{\mathcal{N}}\gamma^\mu \mathcal{N} \partial^2 \left(\bar{\mathcal{N}} \gamma_\mu \mathcal{N} \right)$$

$$\rightarrow - \sum_{k\geq1} \frac{1}{2m^2_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \mathcal{A}_S$$

$$+ \frac{1}{4m^2_{\mathcal{N}}} \sum_{k\geq1} \frac{1}{2m^2_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \left(\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + 3\mathcal{A}_5 - \mathcal{A}_6 \right)$$

$$+ \sum_{k\geq1} \frac{1}{2m^4_{\omega^{(k)}}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 \mathcal{A}_1 \,.$$

- Actually, for example, the non-relativistic four-point contact Lagrangian from the isospin singlet mesons is

$$\mathcal{L}^{singlet} = \mathcal{L}_\omega + \mathcal{L}_f + \mathcal{L}_{\eta'} \,.$$

...interlude...

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Four-point interactions and LECs

- (Isosingplet) By direct comparison

$$\mathcal{L} = \mathcal{L}_\omega + \mathcal{L}_f + \mathcal{L}_{\eta'} \, .$$

with the effective Lagrangian,

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2} C_S \mathcal{A}_S - \frac{1}{2} C_T \mathcal{A}_T \\ & -\frac{1}{2} C_1 \mathcal{A}_1 + \frac{1}{8} C_2 \mathcal{A}_2 - \frac{1}{2} C_3 \mathcal{A}_3 - \frac{1}{8} C_4 \mathcal{A}_4 - \frac{1}{4} C_5 \mathcal{A}_5 - \frac{1}{2} C_6 \mathcal{A}_6 - \frac{1}{16} C_7 \mathcal{A}_7 \,,\end{aligned}$$

- the leading order C_S and C_T are

$$C_S = \sum_{k\geq 1} \frac{1}{m^2_{\omega(k)}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 , \quad C_T = - \sum_{k\geq 1} \frac{1}{m^2_{f(k)}} \left(\frac{N_c g_A^{(k)singlet}}{2} \right)^2$$

- and the LECs of order Q^2 are

$$-\frac{C_1}{2} = \frac{1}{4m^2_{\mathcal{N}}} \sum_{k\geq 1} \frac{1}{2m^2_{\omega(k)}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 + \sum_{k\geq 1} \frac{1}{2m^4_{\omega(k)}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 ,$$

$$\frac{C_2}{8} = \frac{1}{4m^2_{\mathcal{N}}} \sum_{k\geq 1} \frac{1}{2m^2_{\omega(k)}} \left(\frac{N_c g_V^{(k)singlet}}{2} \right)^2 ,$$

$$\cdots$$

$$-\frac{C_7}{16} = \frac{1}{4m^2_{\mathcal{N}}} \sum_{k\geq 1} \frac{1}{2m^2_{f(k)}} \left(\frac{N_c g_A^{(k)singlet}}{2} \right)^2 .$$

- (Isotriplet) $\mathcal{L} = \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{a_1}$.

$$-\frac{1}{2}C_S = -\sum_{k\geq 1}\frac{1}{2m^2_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)^2 + 3\sum_{k\geq 1}\frac{1}{2m^2_{a(k)}}\left(\frac{g_A^{(k)triplet}}{2}\right)^2 ,$$

$$-\frac{1}{2}C_T = -2\sum_{k\geq 1}\frac{1}{2m^2_{a(k)}}\left(\frac{g_A^{(k)triplet}}{2}\right)^2 ,$$

$$-\frac{C_1}{2} = -3\frac{1}{4m^2_{\mathcal{N}}}\sum_{k\geq 1}\frac{1}{2m^2_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)^2 - \sum_{k\geq 1}\frac{1}{2m^4_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)^2$$
$$-\frac{1}{2m_{\mathcal{N}}}\sum_{k\geq 1}\frac{1}{m^2_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)\left(\frac{g_{dV}^{(k)triplet}}{2}\right)$$
$$-\frac{1}{4m^2_{\mathcal{N}}}\sum_{k\geq 1}\frac{1}{2m^2_{a(k)}}\left(\frac{g_A^{(k)triplet}}{2}\right)^2 ,$$

$$\frac{C_2}{8} = -5\frac{1}{4m^2_{\mathcal{N}}}\sum_{k\geq 1}\frac{1}{2m^2_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)^2 - 2\sum_{k\geq 1}\frac{1}{2m^4_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)^2$$
$$-4\frac{1}{2m_{\mathcal{N}}}\sum_{k\geq 1}\frac{1}{m^2_{\rho(k)}}\left(\frac{g_V^{(k)triplet}}{2}\right)\left(\frac{g_{dV}^{(k)triplet}}{2}\right) - 2\sum_{k\geq 1}\frac{1}{2m^2_{\rho(k)}}\left(\frac{g_{dV}^{(k)triplet}}{2}\right)^2$$
$$+\frac{1}{4m^2_{\mathcal{N}}}\sum_{k\geq 1}\frac{1}{2m^2_{a(k)}}\left(\frac{g_A^{(k)triplet}}{2}\right)^2 - 4\sum_{k\geq 1}\frac{1}{2m^4_{a(k)}}\left(\frac{g_A^{(k)triplet}}{2}\right)^2 ,$$

$$-\frac{C_3}{2} = \cdots$$

and so on.

Numerical values

	Original	N_c-Shifted	CD-Bonn	AV-18
C_S $(10^{-4}\,\mathrm{MeV}^{-2})$	1.32	0.976	-0.893	0.0816
C_T $(10^{-4}\,\mathrm{MeV}^{-2})$	0.129	0.342	0.0736	0.125
C_1 $(10^{-9}\,\mathrm{MeV}^{-4})$	-0.225	-0.221	0.436	0.426
C_2 $(10^{-9}\,\mathrm{MeV}^{-4})$	-0.379	-0.951	1.437	1.49
C_3 $(10^{-9}\,\mathrm{MeV}^{-4})$	0.0032	0.0533	0.0240	0.0316
C_4 $(10^{-9}\,\mathrm{MeV}^{-4})$	0.0243	-0.0475	-1.155	-0.304
C_5 $(10^{-9}\,\mathrm{MeV}^{-4})$	-0.353	-0.425	0.799	0.792
C_6 $(10^{-9}\,\mathrm{MeV}^{-4})$	-0.0125	-0.0612	-0.0527	-0.0567
C_7 $(10^{-9}\,\mathrm{MeV}^{-4})$	0.291	0.610	-0.853	-0.536

- CD-Bonn, AV-18 fitting data from resonance saturation model Epelbaum et al. (2002)
- N_c-shifting: considered factor $N_c + 2/N_c$ for $g_A^{(k)triplet}$, $g_{dV}^{(k)triplet}$ etc.
- practical comparing will be meaningful on the stage for studying observables with this set of values

Summary

- Outline of the process

  pre. AdS/CFT, D4/D8/$\overline{\text{D8}}$, holographic baryon, ...

  1. 5d meson and baryon with cubic interactions
  2. down into 4d and carrying out cubic couplings g
  3. integrating out mesons
  4. non-relativistic reduction ($+$constraints)
  5. matching $4N$ operators with LECs

- LECs were derived directly: structures and systematic process
- Some intrinsic problems for these type of approaches
- More relevant and qualitative tests can be done for studying observables (e.g: NN scattering phase shifts)