Nuclear symmetry energy and Neutron Star Cooling

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Nuclear Symmetry energy

- Large uncertainties in nuclear physics at high density ($\rho > \rho_0$)
- Energy per baryon

$$e(\rho, x) = e(\rho, 1/2) + S_2(\rho)(1-2x)^2 + \cdots$$

- The symmetry energy parameter

$$S_v = S_2(\rho_0), \quad L = 3\rho_0 (dS_2/d\rho)_{\rho_0},$$
$$K_{sym} = 9\rho_0^2 (d^2 S_2/d\rho^2)_{\rho_0}, \quad Q_{sym} = 27\rho_0^3 (d^3 S_2/d\rho^3)_{\rho_0}$$

- The density dependence of the symmetry energy in nuclear astrophysics

  - The neutronization of matter in core-collapse supernovae

  - The radii and crust thinckness of neutron stars

  - The cooling rate of neutron stars

  - The r-process nucleosynthesis

We can estimate the range of symmetry energy from experiments and theories.

- Nuclear Mass fitting
  Liquid droplet model, Microscopic nuclear force model (Skyrme force model)
- Neutron Skin Thickness
  ^{208}Pb, Sn with RMF and Skyrme
- Dipole Polarizabilities
- Heavy Ion Collisions
- Neutron Matter Theory
  Quantum Monte-Calro, Chiral Lagrangian
- Astrophysical phenomenon
  Neutron stars mass and radius

The optimal points for S_v and L

- S_v and L from nuclear mass constraints

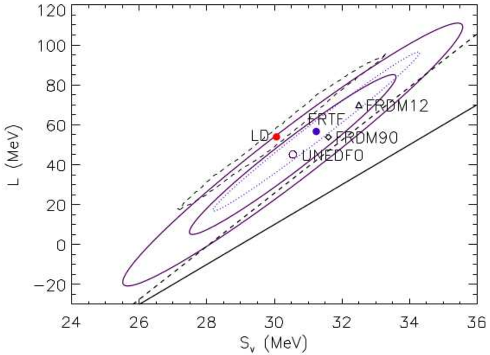


Figure: from J.M.Lattimer and Y. Lim, arXiv:1203.4286

- S_v and L are in different points but LDM, FRTF, HF give the same slope.

The overlapped area for S_v and L

- S_v and L from nuclear interactions

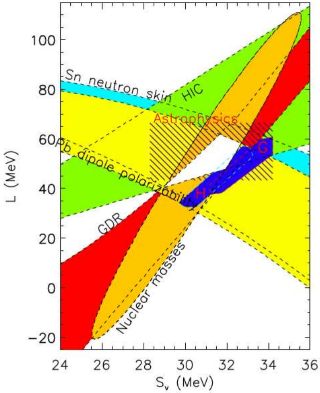


Figure: from J.M.Lattimer and Y. Lim, arXiv:1203.4286

- S_v and L are in the range $29.5 - 32.7$ MeV and $42 - 62$ MeV.

The relativistic equations of thermal evolution

- Diffusion equation

$$\frac{1}{4\pi r^2 e^{2\Phi}}\sqrt{1-\frac{2Gm}{c^2 r}}\frac{\partial}{\partial r}(e^{2\Phi}L_r) = -Q_\nu - \frac{C_v}{e^\Phi}\frac{\partial T}{\partial t} \tag{1a}$$

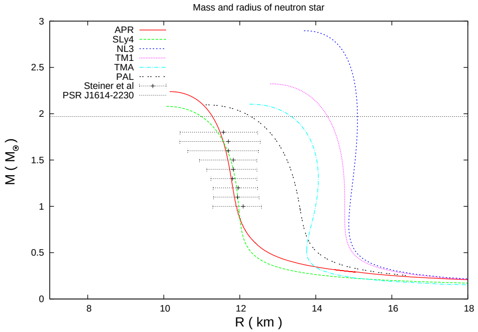
$$\frac{L_r}{4\pi r^2} = -\kappa\sqrt{1-\frac{2Gm}{c^2 r}}e^{-\Phi}\frac{\partial}{\partial r}(Te^\Phi) \tag{1b}$$

- Q_ν : Neutrino emission rate : $Q_\nu = Q_\nu(T, \rho_n, \rho_p)$
- C_v : Heat capacity (specific heat) : $C_v = C_v(T, \rho_n, \rho_p)$
- κ : Thermal conductivity : $\kappa = \kappa(T, \rho_n, \rho_p)$
- e^Φ : General relativistic metric function : $e^\Phi = \sqrt{1-\frac{2GM}{rc^2}}$
- L (T) is defined on even (odd) grid : L_{2i}, T_{2i+1}
- Two boundary conditions : $L_0 = 0$, $T_s = T_s(T_b)$
- Henyey method is used to find new temperature

$$T_i^{n+1} = T_i^n + \Delta t\frac{dT_i^n}{dt} \quad \rightarrow \quad T_i^{n+1} = T_i^n + \Delta t\frac{dT_i^{n+1}}{dt}$$

Equation of state

- Lots of nuclear force models are available
- EOS based on RMF : TM1, FSU Gold, NL3, . . .
- EOS based on variational principle : APR (The most accurate)
- EOS based on EDF : SLy4 (0, $\cdots$, 10)
- Phenomenological model : PAL
- EOS should explain the maximum mass of neutron stars greater than 1.97$M_\odot$



Construction of EOS

- Neutron star is cold after 30s $\sim$ 60s of its birth
  We calculate NS structure using TOV. No need to think convection

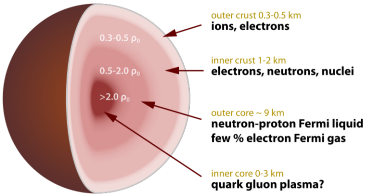


Figure: Neutron star inner structure (from Wikipedia)

- Core (Inner, outer core)
  Only uniform matter exists. Hyperons or quark matter
- Inner crust
  Heavy nuclei + free gas of neutrons + free electrons
- Outer crust
  Heavy nuclei + free electrons (No dripped neutrons)

Physics from EOS

- Nuclear matter in the core of neutron stars
  Easy to calculate (Relativistic, Non-relativistic)
- Three approaches for crust : Liquid droplet(O), Thomas Fermi(O), Hartree-Fock(x)
  Among the many EOSs, APR, SLy series are probably the best
- Composition of constituents : Protons, neutrons, electrons
  On and off direct URCA process (proton fraction)
- Boundaries of inner crust and out crust, (neutron drip)
  Boundaries of URCA process
- Atomic number in case of crust
  Need to calculate Q_ν, C_V, and κ
- Effective masses for proton and neutron
  Effective masses are involved in formulae for Q_ν, C_V, and κ
- Volume fraction of heavy nuclei

NS Crust

- In NS crust, heavy nuclei exist with free gas of neutrons and electrons
- Liquid droplet - energy minimization : analytic

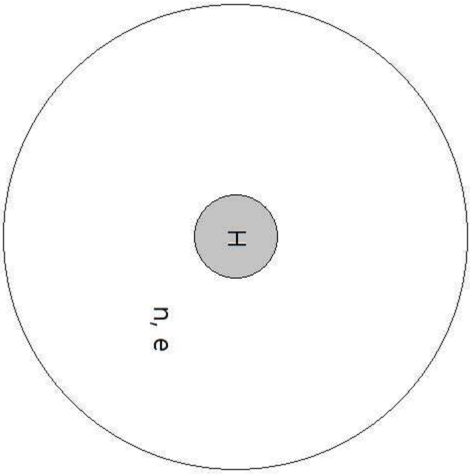
$$F = u n_i f_i + \frac{3s(u)}{r_N}(\sigma(x) + \mu_s \nu_n) + \frac{4\pi}{5}(r_N n_i x_i e)^2 c(u)\,.$$

  Easy to deal with all 3D phases
- Thomas Fermi - energy minimization : numerical

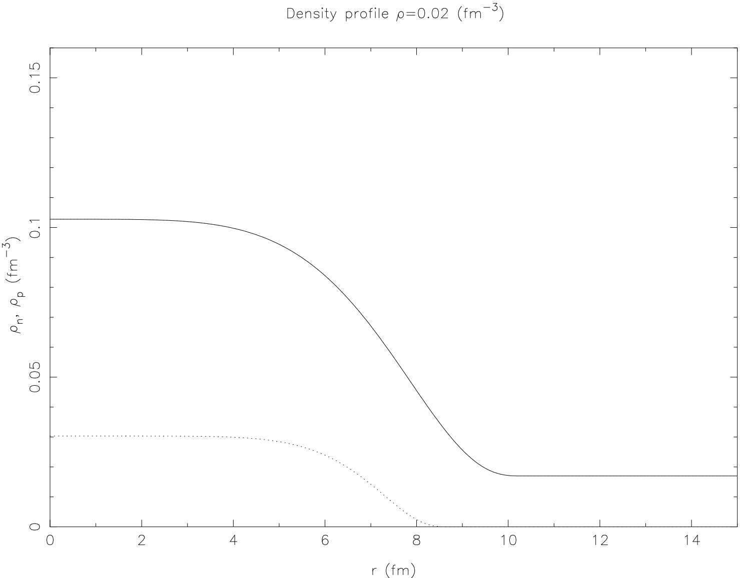
$$\rho_i(r) = \begin{cases} (\rho_i^{\rm in} - \rho_i^{\rm out})\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + \rho_i^{\rm out}\,, & r < R_i \\ \rho_i^{out}\,, & r \geq R_i\,. \end{cases}$$

  Plug the density profile into energy functional and integrate to get energy
  Takes much longer than LDM, hard to deal with nuclear pasta phases.
- Need to develop Hartree-Fock code

- Schematic figure for LDM



- Result from TF



Neutrino emission in the core

- Neutrino emission rates (Q_ν) in the core (erg/s/cm^3)

$$
\begin{aligned}
&\text{Direct URCA} \begin{cases} n \to p + e^- + \bar{\nu_e} \\ p + e^- \to n + \nu_e \end{cases} && \sim 10^{27} T_9^6 \\
&\text{Modified URCA} \begin{cases} n + n' \to n' + p + e^- + \bar{\nu_e} \\ n' + p + e^- \to n' + n + \nu_e \end{cases} && \sim 10^{20} T_9^8 \\
&\text{Nucleon Bremsstrahlung} \left\{ N_1 + N_2 \to N_3 + N_4 + \nu + \bar{\nu} \right. && \sim 10^{19} T_9^8 \\
&K\text{-condensate} \begin{cases} n + K^- \to n + e^- + \bar{\nu_e} \\ n + e^- \to n + K^- + \nu_e \end{cases} && \sim 10^{24} T_9^6 \\
&\pi\text{-condensate} \begin{cases} n + \pi^- \to n + e^- + \bar{\nu_e} \\ n + e^- \to n + \pi^- + \nu_e \end{cases} && \sim 10^{26} T_9^6
\end{aligned}
\tag{2}
$$

Formulae for neutrino emission

- $Q_{\rm dir} = 4.0 \times 10^{27} \frac{m_n^* m_p^*}{m_n m_p} \left(\frac{n_e}{n_0}\right)^{1/3} T_9^6 \mathcal{R}_{\rm dir}$ erg cm^{-3}s^{-1}
- $Q_{\rm mod,n} = 8.55 \times 10^{21} \left(\frac{m_n^*}{m_n}\right)^3 \left(\frac{m_p^*}{m_p}\right) \left(\frac{n_p}{n_0}\right)^{1/3} T_9^8 \alpha_n \beta_n \mathcal{R}_{\rm mod,n}$ erg cm^{-3}s^{-1}
- $Q_{\rm mod,p} = 8.53 \times 10^{21} \left(\frac{m_n^*}{m_n}\right) \left(\frac{m_p^*}{m_p}\right)^3 \left(\frac{n_p}{n_0}\right)^{1/3} T_9^8 \alpha_p \beta_p \mathcal{R}_{\rm mod,p}$ erg cm^{-3}s^{-1}
- $Q_{nn} = 7.4 \times 10^{19} \left(\frac{m_n^*}{m_n}\right)^4 \left(\frac{n_n}{n_0}\right)^{1/3} T_9^8 \alpha_{nn} \beta_{nn} \mathcal{N}_\nu \mathcal{R}_{nn}$ erg cm^{-3}s^{-1}
- $Q_{pp} = 7.4 \times 10^{19} \left(\frac{m_p^*}{m_p}\right)^4 \left(\frac{n_p}{n_0}\right)^{1/3} T_9^8 \alpha_{pp} \beta_{pp} \mathcal{N}_\nu \mathcal{R}_{pp}$ erg cm^{-3}s^{-1}
- $Q_{np} = 1.5 \times 10^{20} \left(\frac{m_n^* m_p^*}{m_n m_p}\right)^2 \left(\frac{n_p}{n_0}\right)^{1/3} T_9^8 \alpha_{np} \beta_{np} \mathcal{N}_\nu \mathcal{R}_{np}$ erg cm^{-3}s^{-1}

Neutrino emission in the crust

- Neutrino emission rates (Q_ν) in the crust (erg/s/cm^3)

$$\begin{aligned}
&\text{Electron - nucleus bremsstrahlung} : e + (A, Z) \rightarrow e + (A, Z) + \nu + \bar{\nu} \\
&\text{Nucleon Bremsstrahlung} : N_1 + N_2 \rightarrow N_3 + N_4 + \nu + \bar{\nu} \\
&\text{electron positron annihilation} : e^- + e^+ \rightarrow e^- + e^+ + \nu + \bar{\nu} \\
&\text{Plasmon decay} : \gamma \rightarrow \nu + \bar{\nu}
\end{aligned} \tag{3}$$

Heat capacity in the core and crust

- Heat capacity in the core
  Degenerated non-relativistic baryons and degenerate relativistic electrons (leptons)

$$C_v^{\text{core}} = C_n^{\text{core}} + C_e \tag{4}$$

  where

$$C_n^{\text{core}} = \frac{k_B^2}{3\hbar^3} T\left(m_n^* p_F(n_n) + m_p^* P_F(n_p)\right), \quad C_e = \frac{k_B^2}{3\hbar^3} T m_e^* p_F(n_e) \tag{5}$$

- Heat capacity in the crust
  Heat capacity has the contribution from free neutrons, electrons, and ions

$$C_v^{\text{crust}} = C_n^{\text{crust}} + C_e + C_i^{\text{crust}} \tag{6}$$

  where

$$C_n^{\text{crust}} = \frac{k_B^2}{3\hbar^3} T m_n^* p_F(n_n) \tag{7}$$

Thermal conductivity

- Core : electrons + neutrons
- Crust : electrons
- Main contribution to thermal conductivity comes from electrons
- Superfluidity gives reduction factors for neutrino emissivity, heat capacity, and thermal conductivity

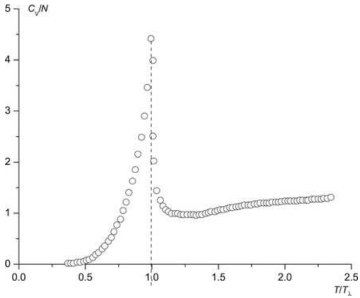
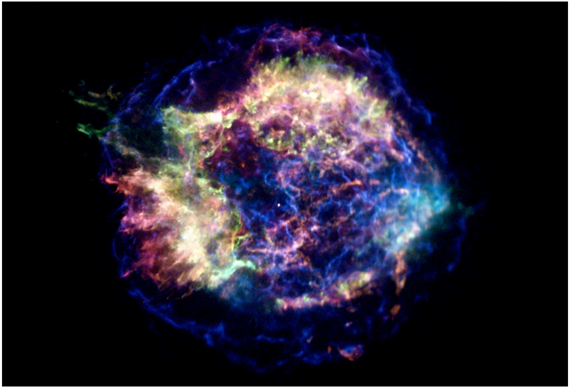


Figure: ^{4}He heat capacity

Cas A Neutron Star

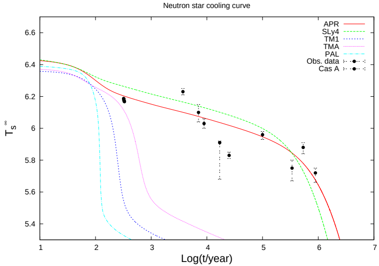
- Casiopea A supernova remnant

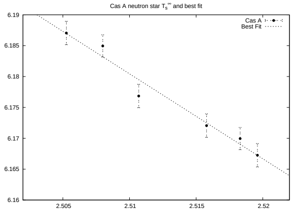


- First light : Chandra observations (1999)
- Distance : $\sim$ 3.4 kpc
- Age : 330$\pm$ 10 years

NS Cooling (Standard)

- Standard cooling : DU, MU, NB, No superfluidity, No bose condensation



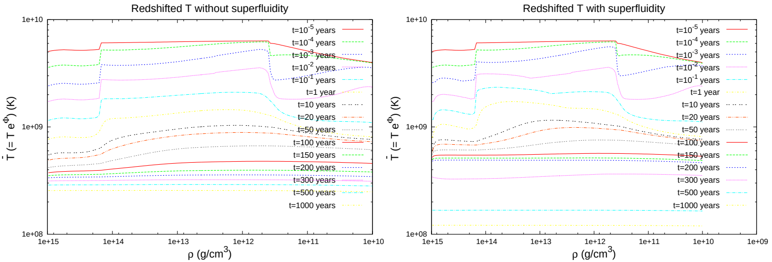


- Direct URCA process turned on in case of TM1, TMA and PAL
- Standard cooling process cannot explain the fast cooling in Cas A

$$\frac{d\ln T_s^\infty}{d\ln t} = -1.375 \quad \text{(best fit)} \quad \text{vs} \quad -0.07 \sim -0.15 \quad \text{(standard cooling)}$$

Thermal Relaxation

- Redshited temperature profile for given time



- Isothermal phase is achieved within 300 years

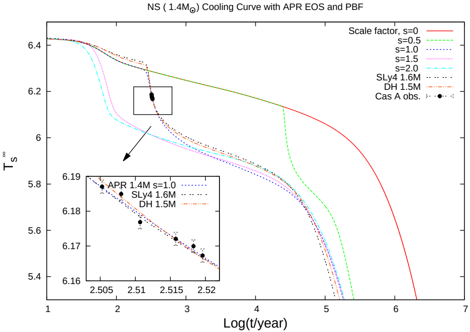
Nuclear Superfluidity

- Nuclear ground state can be achived with Cooper pairing
  ex) Even-Even nuclei is more stable than even-odd, odd-odd
- Nuclear superfluidity is uncertain at high densities ($\rho \sim \rho_0$).
  $^3P_2$ N, $^1S_0$ P for higher densities (Neutron star core)
  $^1S_0$ N for lower densities (Neutron star crust)
- Nuclear superfluidity gives reduction factor for neutrino emission, heat capacity, and thermal conductivity.
  $Q_{DU} \rightarrow Q_{DU}R_{DU}$, $Q_{Mod,U} \rightarrow Q_{DU}R_{Mod,U}$, $Q_B \rightarrow Q_B R_B$
  $C_{n,p} \rightarrow C_{n,p}R_{n,p}$, $C_n^{\mathrm{crust}} \rightarrow C_n^{\mathrm{crust}}R_n$, $\kappa_n^{\mathrm{core}} \rightarrow \kappa_n^{\mathrm{core}}R_{\kappa,n}$
- It opens neutrino emission process from Cooper Pair Breaking and Formation (PBF).

$$Q_{PBF} = 1.17 \times 10^{21} \frac{m_n^*}{m_n} \frac{p_F(n_n)}{m_n c} T_9^7 \mathcal{N}_\nu \mathcal{R}(T/T_{c,n}) \,\mathrm{erg\,cm^{-3}\,s^{-1}}$$

NS Cooling (Enhanced)

- Enhanced cooling : Pair Breaking and Formation (PBF)



- Observation of Cas A NS indicates the existence of nuclear superfluidity
- Different EOSs (APR, SLy4) may give same slope but different critical temperature for nuclear matter
- EOSs good for Cas A cooling are passing SLB criteria area.

Conclusion

- NS Cooling is to solve partial differential equation numerically.
- There are a lot of nuclear equation of state but only a few of them are good.
- Fast cooling in Cas A NS can be explained with nuclear superfluidity and PBF
- T_c for $3P_2$ is $5 \sim 7 \times 10^8$K.
- The range of S_v and L can be confirmed from Cas A cooling rate.